Hygromecanical Modeling of Adhesive Line in Laminated Wood Composite

INTRODUCTION

In wood products design, dimensional stability is of primary importance especially in layered wood composites where a small deformation can result in poor behavior. This is particularly true for appearance products such as parquetry, flooring, cabinetry and furniture components. Non-homogeneous adsorption and desorption of water vapor by the composite may induce cupping, and consequently decrease product value. For appearance products, a small deformation in the material can result in an undesirable performance. In this context, the design of stable product is of primary importance for the end value of the composite. Finite element method has been successfully used in engineered wood flooring (EWF) (Blanchet et al.) but glue line modeling appeared has critical in the phenomenon involved.

The objective of this paper is to present the importance of the glue line in layered wood composite using the finite element method. Mesh density, interpolation level, mesh adaptation and linear interpolation of the adhesive properties were used in EWF model to demonstrate the critical role of the adhesive. First, the problem is described. Then, the mathematical model and its discretization are presented. In the last section, the parameters from a previous study were applied to the model of a EWF construction. The results are presented in the fourth section. Finally, some conclusions are presented in the last section of the paper.

PROBLEM STATEMENT

The flooring strip considered in this study is the same as described in Blanchet et al WWWW. This construction is briefly recalled here. The EWF is a free standing three layer strip (Figure 1). It is 65 mm-wide, 14 mm-thick and 1000 mm long. The construction considered is a 4 mm-thick sugar maple plank as surface layer (SL), an 8 mm-thick white birch core layer (CL) and a 2-mm thick yellow birch veneer as backing layer (BL). The core layer is made of 22 mm-wide sticks with 2 mm spacing. UF resin is used to bond layers together.

The deformation is assumed to be caused by water vapor desorption from 6.3 to 5 percent occurring by convection from the top surface only. This corresponds to a decrease from 50 to 20 percent RH at 20ºC. All other edges and the bottom surface are assumed impervious. Each wood layer of the composite is assumed orthotropic and elastic, no mechano-sorptive effects were taken into account as previous work demonstrated that the behavior is elastic under the conditions considered in this study (Blanchet *et al*. 2003b). Since the components were conditioned before and after assembly, the flooring is assumed to be initially free of stress.

The modeling of the behavior of engineered flooring strips requires the knowledge of the physical and mechanical properties of the different wood species used for each layer as well as of the UF resin. The material properties used in the current study were take in Blanchet et al. and are recall in Table 1.

MODELING APPROACH

Mathematical Model

Mathematical Model

The governing equation used for the description of the mechanical aspect of the problem is the three-dimensional equation of equilibrium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \qquad (1)$$

where body forces are assumed to be negligible. σ_{ij} are the normal and shear stress components, expressed in the rectangular coordinate system. Because the wood material and the UF resin are assumed elastic, Hooke's law can be used to relate stresses to strains.

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \beta_{kl}\Delta M) \tag{2}$$

where C_{ijkl}: stiffness tensor; ε_{kl}: normal and shear strain components; γ_{ij}: engineering shear strain components; β_{kl}: moisture shrinkage/swelling coefficients ($\%^{-1}$); ΔM: moisture content change (%). Because the UF resin is assumed isotropic, Hooke's law can be simplified for that component of the composite by assuming: $E_1= E_2= E_3=E$, $\nu_{23}= \nu_{32}=\nu_{13}= \nu_{31}=\nu_{12}= \nu_{21}= \nu$; $G_{23}= G_{13}= G_{12}=E/2(1+\nu)$; $\beta_1=\beta_2=\beta_3=\beta$.

The normal and shear strains are related to the displacements, u_1, u_2, and u_3 measured along the x_1, x_2, and x_3 directions, respectively.

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

The transient moisture movement through the model is described by the three-dimensional moisture conservation equation.

$$\frac{d_b}{100}\frac{\partial M}{\partial t} + \vec{\nabla}\cdot\left(-K_M\,\vec{\nabla} M\right) = 0 \qquad \text{with } K_M = \frac{D\,d_b}{100} \tag{4}$$

where K_M: is the tensor of effective conductivity (kg m^{-1} s^{-1} $\%^{-1}$); D: the tensor of effective diffusion (m^2 s^{-1}) and d_b : basal density (kg m^{-3}).

Discretization of the Mathematical Model

The finite element modeling of hygromechanical distortion in EWP was performed using the post-processing software VU and finite element code MEF++. The finite element discretization of the Galerkin weak form of the mechanical equilibrium equation (2) and moisture conservation equation (4) was performed using standard isoparametric and linear or quadratic interpolation of the unknown displacements u_1, u_2, u_3, and moisture, M. Essential boundary conditions correspond to specified values of the displacements, u_1, u_2, and u_3, or moisture content, M. Natural boundary conditions correspond to specified values of the normal stress vector or moisture flux. The time discretization of the equation is accomplished with the standard Euler implicit time marching scheme. The predicted values of M, u_1, u_2, and u_3 depend on position and time. A single coupled system of discrete equations is solved for the displacements and M at each time step. A user-specified initial time increment of 0.5 second is specified. The following time increments are automatically adjusted between 0.1 and 100000 seconds by the MEF++ software based on the convergence rate.

Application of the Discretized Model to Engineered Wood Flooring

The proposed model was used to shed light on the hygromecanic behavior of the glue line. The model is central part of an engineered wood flooring strip as shown in Figure 1. The domain considered corresponds to half the width of the strip and a length of 24 mm as presented in Figure 2.

The principal material directions of wood are assumed to be perfectly oriented with the strip and wood growth rings are assumed perfectly flat. As a result, a Cartesian coordinate system can be used. The tangential, longitudinal, and radial properties of wood are specified for both surface and backing layers in the x_1-, x_2-, and x_3-directions, respectively. In the core layer, the x_1-, x_2-, and x_3-directions correspond to the longitudinal, radial and tangential wood directions respectively as this layer is cross grain oriented compare to the surface and backing layers. A perfect adhesion between wood and the UF resin was considered in the glue line. The model takes into account the hysteresis in the adsorption and desorption isotherms according to the results of Goulet and Fortin (1975).

We used a series of 5 finite element meshes in this paper (see Table 2 for details of the meshes). A coarse finite element mesh uses 1 mm-thick elements the surface and core layers and 2 mm-thick elements for the backing layer. The 0.1 mm thick superior glue line elements was meshed with one, two, three and six elements. The finer mesh is used to compute a reference solution; this solution is used to compare and evaluate the various solutions obtained with the other meshes.

Adaptative meshing of the model

VA-Y ANDRÉ

Initial and boundary conditions

Initial and boundary conditions must be specified for moisture transfer as well as the mechanical components of the model. The initial condition for moisture content was as follow:

$$M(x_1,x_2,x_3,t_0) = M_0 = 6.3\ \% \qquad \forall\ (x_1, x_2, x_3) \quad (5)$$

where M_o is the initial moisture content.

The moisture transfer boundary conditions specify no moisture flux through any surface except the top one through which desorption is assumed to occur. On that surface, the moisture flux is given as follows:

$$q = h\ (M - M_\infty) \qquad (6)$$

where h is the convective mass transfer coefficient (kg m^{-2} s^{-1} $\%^{-1}$) and M_∞ is the equilibrium moisture content (%).

The following initial and boundary conditions were used for the mechanical part of the model. The initial conditions correspond to the fact that the flooring strip is initially assumed stress free and undeformed. The essential boundary conditions are illustrated in Figure 2. They are defined as follows:

$$u_1 = 0 \text{ at } (32.5, x_2, x_3) \quad (7)$$

$$u_2 = 0 \text{ at } (x_1, 0, x_3) \text{ and } (x_1, 24.0, x_3) \quad (8)$$

$$u_3 = 0 \text{ at } (32.5, x_2, 0) \quad (9)$$

Equation (7) creates a symmetry axis along the centerline of the strip. Equation (8) specifies that the domain considered cannot elongate since it is part of a longer strip. This corresponds to the restraint that would be provided longitudinally by the remaining part of the flooring strip which is not modeled. Finally the strip is freestanding; this adds an inequality boundary condition (contact condition) to our model. However using the symmetry of the mechanical component of the problem we can circumvent the more complicated contact condition that should be applied to the problem. Equation (9) specifies that the bottom part of the strip cannot move vertically along the x3 axis. The model was introduced in a previous study (Blanchet *et al*. 2005).

Interpolation of the wood adhesive interface

As the glue line appears to be an important component of the desorption phenomenon in the product, it seems to be a coarse approach to only a single layer of element in the glue line. It is especially true taking into account and important variation of the physical (diffusion coefficient) and mechanical properties from wood to adhesive. In a model using six elements in the glue line, physical and mechanical properties were linearly interpolated from wood to adhesive properties in the first two elements to reflect the properties of the wood-adhesive interface. Then the adhesive properties were used in the central layers of element in the glue line. Finally, properties were linearly interpolated from adhesive to wood properties in the last two layers of element of the glue line. This approach is graphically described in the Figure 3.

Results and Discussion

As a comparative tool for the different solutions we use a measure of the cupping. Since we are in a situation where symmetry applies (for the mechanical component of the problem) we have a simple definition of cupping. We introduce the points A and B (see Figure 2) and we can define cupping C(t,A,B) as the evolution in time of the difference of vertical displacement of the points A and B:

$$A=(0,12,14.2) \qquad B=(32.5,12,14.2) \qquad C(t,A,B)=u_3(t,A)-u_3(t,B)$$

Introducing U_{Ref} the linear displacements obtained with Mesh_ref. and $C_{Ref}(t,A,B)$ the associate cupping we define the relative error (%) (with respect to cupping) as

$$error=\frac{100*(C(t,A,B)-C_{Ref}(t,A,B))}{C_{Ref}(t,A,B)}$$

and if we have C^1 and C^2 two different cupping obtained from two displacements U^1 and U^2 we define the variation of cupping (%) $\Delta C_{1,2}$ as

$$\Delta C_{1,2}=\frac{100*(U^1(t,A,B)-U^2(t,A,B))}{U^2(t,A,B)}$$

Since cupping is one of the mechanical phenomenons of interest, this choice of comparative measure is appropriate. Finally, since we have an unsteady problem, we have to fix a time interval for our purposes. In this case we chose to study the phenomenon for 82 days.

Single element layer in the glue line

Previous work has shown the sensitivity of the glue line in modeling of EWF (Blanchet et al.). In this work, a single layer of element was used to model the glue line. Such an important phenomenon in the glue line should be investigated at a higher level of interpolation. Figure 4 presents the effect of the interpolation level of the displacements on the cupping. In this figure, we compare a linear and a quadratic interpolation of the displacements for two meshes: one with a single layer in the glue line and with three elements in the glue line. As can be seen in the top left figure and at the bottom there is little difference between the cupping for the linear and quadratic interpolation for either diffusions. We have a variation of less than 5% between the two responses, which seems to indicate that in either cases (1-layer or 3- layers mesh) a linear interpolation of the displacements is sufficient: using a richer interpolation will not give an important gain in accuracy (keeping in mind that our criteria is the cupping). However as the top right figure shows we have an important loss of accuracy as the diffusion decrease, going from an error of less than 10% for a diffusion of 1.0×10^{-14} to an error of more then 20% for the first 20 days for a diffusion of 5.0×10^{-16}. This graph was obtained by comparing the linear and quadratic solution to U_{Ref}, the linear solution obtained with the finest mesh (Mesh_ref in Table 2). We can summarize our observations as follow

- As the effective diffusion of the resin diminishes the gap between the linear and quadratic interpolation increase, it is however relatively small

- As we increase the number of layers in the glue line the gap between the linear and quadratic interpolation increase, it is relatively small
- For the 1-layer mesh as the diffusion diminishes we have an important loss of accuracy which cannot be corrected by using an interpolation of higher degree.

And the effects of multiple layers meshes has to be investigate

Single element layer vs. multiple layers of element

The binder in this model has a very small value of conductivity. Our purpose here is to illustrate the importance of a good geometrical description of the glue line considering the small value of conductivity. For this, we used a series of decreasing values for its effective diffusion coefficient: 1.0×10^{-14}, 1.0×10^{-15}, 5.0×10^{-16}, 3.0×10^{-16}, 1.0×10^{-16} ($m^2\ s^{-1}$). For each value we computed the cupping using two meshes: a coarse one with one layer of hexahedral elements in the glue line and a finer one with 3 elements (Mesh_1 and Mesh_3).

Figure 5 shows the important difference between the solutions obtained with a 1-layer and a 3-layers mesh. Even though we had a very simple problem (constant value for all properties) we managed to produce important variation between the two solutions for each value of diffusion. As the diffusion increase the variation seems to diminish. However even for relatively high value of effective diffusion we have an important gap, up to 10% (see Figure 6, top graph) between the 1-layer and the 3-layers solution in the "transition" phase of the cupping. The discrepancy between the two solutions for each value of diffusion can be explained by the poor approximation of the mechanical phenomenon in the glue line. A fortiori for a problem with elaborate and complex relation between properties we should use a very fine mesh in the glue line.

We already showed that the degree of interpolation for the displacements has a very small role in the behavior of the cupping. Figure 6 shows the importance of the number of layers of elements in the glue line. The top graph summarizes the variation of cupping from 1 to 3 layers mesh for diffusion varying from 1.0 x 10^{-16} to 1.0 x 10^{-14}. The two bottom graphs show the error to the reference solution. On the left we have plotted the error for a linear interpolation of the displacements for an effective diffusion of 5.0 x 10^{-16} for three meshes: Mesh_1, Mesh_2, Mesh_3 (see Table 2). From the error for the 2 and 3 layers meshes we conclude that a single layer of element is oversimplifying the geometry and neglects some important effects due to the mechanical properties of the glue film. Moreover it shows that a linear interpolation as a tendency to under estimate the solution. On the right we show the error for a quadratic interpolation with various effective diffusion of the glue. In this case we have a under estimation of the solutions. Comparing with the error for a 1-layer mesh (Figure 4, top right) it is clear that a mesh with 3 layers of elements in the upper glue line must be used.

From this theoretical work on the finite element solution of the model a number of important points can be noted

- The degree of interpolation for the mechanical component of the model is of minor effect. The choice of degree interpolation higher then linear is unnecessary (see Figure 4).
- The use of a mesh with a single element in the glue line can lead to important error in approximation (see Figure 6). This is particularly true for small values of effective diffusion for the glue.
- The use of complex meshing tools such as adaptative meshing seems superfluous at least for simple properties such as studied here.

Adaptative meshing

Even though we have small values of effective diffusion in the glue line we produce solutions which are relatively different. We expected the solutions to be almost identical for such small diffusion values. This raises question on the hypothesis regarding the description of the glue line and lead to a more refined model for the glue line. Specifically we introduced two interfaces. Those interfaces can be viewed as transition (or mixture) zones between the wood and the glue.

Linearly interpolated properties

Let D_0, D_1 and D_G be the effective diffusion for wood species 0, 1 and for the glue line between them respectively. Let us denote by z_0, z_1, z_2 and z_3 the heights of different parts of the glue line (see Figure 3). We introduce the functions

$$\lambda_0(x_3) = \begin{cases} 0 & x_3 \le z_0 \\ \dfrac{(x_3 - z_0)}{(z_1 - z_0)} & x_3 \in [z_0, z_1] \\ 1 & x_3 \ge z_1 \end{cases} \qquad \lambda_2(x_3) = \begin{cases} 1 & x_3 \le z_2 \\ \dfrac{(x_3 - z_3)}{(z_2 - z_3)} & x_3 \in [z_2, z_3] \\ 0 & x_3 \ge z_3 \end{cases}$$

and we construct the linear interpolation $\hat{D}_G$ for the effective diffusion of the glue as

$$\hat{D}_G(x_3) = \begin{cases} \lambda_0(x_3)D_G + (1 - \lambda_0(x_3))D_0 & x_3 \le z_2 \\ \lambda_2(x_3)D_G + (1 - \lambda_2(x_3))D_1 & x_3 \ge z_2 \end{cases}$$

Using this construction, the effective diffusion for the glue line will be a linear combination of the diffusion going from the wood specie 0 diffusion (at z_0) to the glue diffusion (at z_1). Then from z_1 to z_2 the diffusion will correspond to the glue diffusion. And finally from z_2 to z_3 we will have a linear combination going from pure glue to wood specie 1.

Table 1 Finite element model parameters

Parameters	Material			
	Surface	Core	Backing	Binder
	Sugar Maple	White Birch	Yellow Birch	UF resin
d_b (kg m^{-3})	[1]597	[1]559	[1]506	[4]1500
D (m^2 s^{-1})				[5]1.0 x 10^{-14}
D_l (m^2 s^{-1})	[2]2.2 x 10^{-9}	[2]2.2 x 10^{-9}	[2]2.2 x 10^{-9}	
D_R (m^2 s^{-1})	[5]1.8 x 10^{-11}	[2]4 x 10^{-11}	[2]4 x 10^{-11}	
D_T (m^2 s^{-1})	[5]1.8 x 10^{-11}	[2]4 x 10^{-11}	[2]4 x 10^{-11}	
M_0 (%)	6.3	6.3	6.3	6.3
M_∞ (%)	5	5	5	5
h (kg m^{-2} s^{-1} %$^{-1}$)	[2]3.2 x 10^{-4}	[2]3.2 x 10^{-4}	[2]3.2 x 10^{-4}	[2]3.2 x 10^{-4}
β (m m^{-1} %$^{-1}$)				[5]1.9 x 10^{-2}
β_L(m m^{-1} %$^{-1}$)	[6]1.5 x 10^{-4}	[1]1.5 x 10^{-4}	[1]1.5 x 10^{-4}	
β_R(m m^{-1} %$^{-1}$)	[6]2.1 x 10^{-3}	[1]1.7 x 10^{-3}	[1]1.9 x 10^{-3}	
β_T(m m^{-1} %$^{-1}$)	[6]3.3 x 10^{-3}	[1]2.4 x 10^{-3}	[1]2.3 x 10^{-3}	
α_L(m m^{-1} %$^{-1}$)	[6]1.8 x 10^{-4}			
α_R(m m^{-1} %$^{-1}$)	[6]1.9 x 10^{-3}			
α_T(m m^{-1} %$^{-1}$)	[6]2.8 x 10^{-3}			
E (GPa)				[4]9
E_L (GPa)	[3]13.810	[3]12.045	[3]15.251	
E_R (GPa)	[3]1.311	[3]1.069	[3]1.251	
E_T (GPa)	[3]0.678	[3]0.516	[3]0.641	
G_{LR} (GPa)	[3]1.013	[3]0.829	[3]0.971	
G_{RT} (GPa)	[3]0.255	[3]0.200	[3]0.242	
G_{LT} (GPa)	[3]0.753	[3]0.607	[3]0.721	
ν				[4]0.35
ν_{LT}	[3]0.50	[3]0.43	[3]0.45	
ν_{RT}	[3]0.82	[3]0.78	[3]0.70	
ν_{TL}	[3]0.025	[3]0.018	[3]0.018	
ν_{RL}	[3]0.044	[3]0.043	[3]0.035	
ν_{TR}	[3]0.42	[3]0.38	[3]0.36	
ν_{LR}	[3]0.46	[3]0.49	[3]0.43	

[1]Jessome (2000), [2] Siau (1995), [3]Bodig et Jayne (1973), [4]Dorlot et al. (1986), [5]Blanchet et al. (2005) , [6]Goulet et Fortin (1975)

L : longitudinal ; R : radial ; T : tangential

Table 2 Number of elements and nodes of the various meshes used..

	Number of layers of elements					Total number of elements/ nodes
	sugar maple surface	glue line top	white birch core	glue line bottom	yellow birch backing	
Mesh_1	4	1	8	1	1	2328 / 3029
Mesh_2	4	2	8	1	1	2496 / 3224
Mesh_3	4	3	8	1	1	2664 / 3419
Mesh_ref	12	6	24	6	6	33984 / 38150

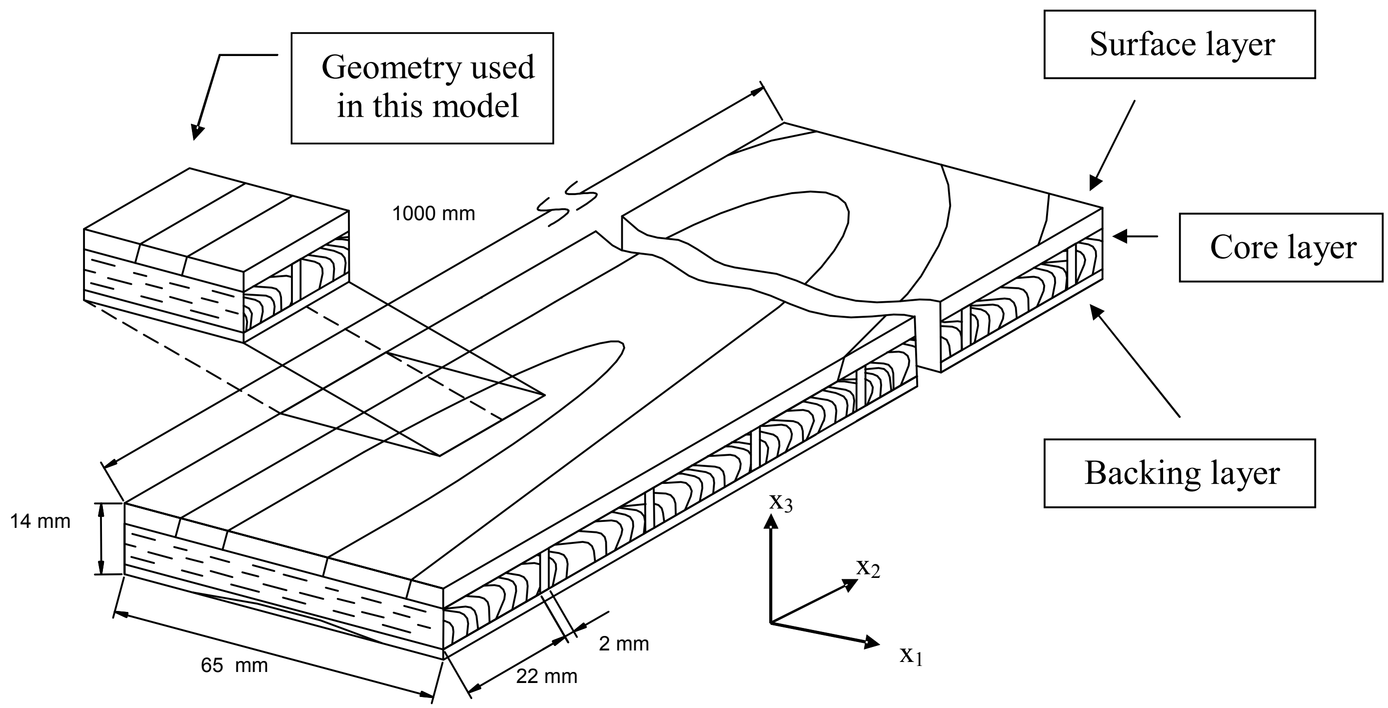


Figure 1 Engineered wood flooring construction and the geometry used in the model.

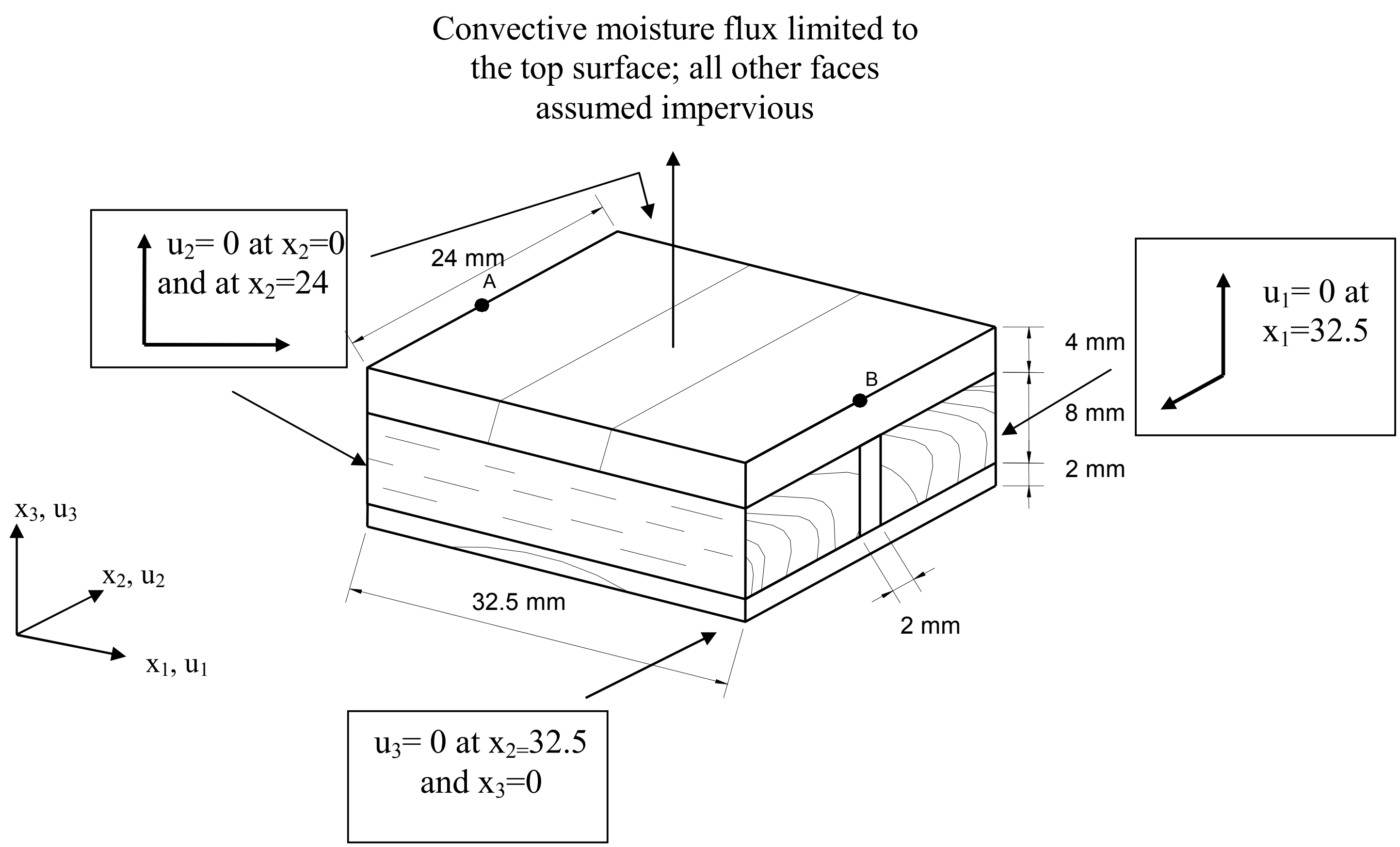


Figure 2 Boundary conditions applied to the geometry of the model domain considered.

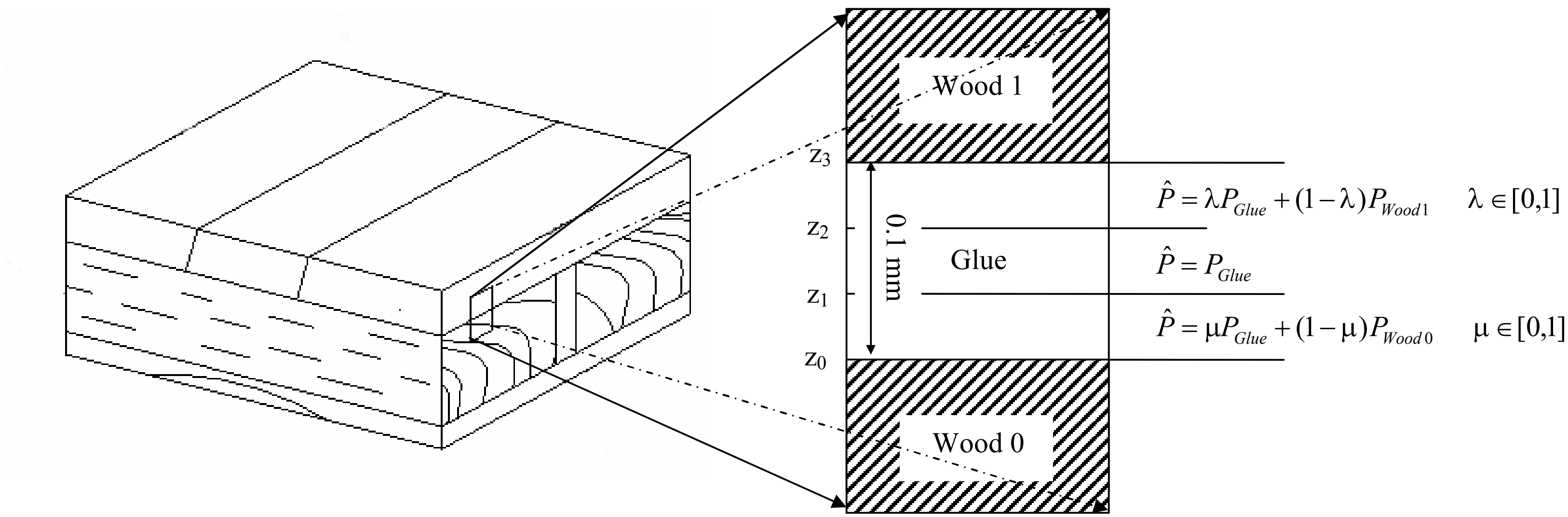


Figure 3 Schematic of the linear interpolation of a property P in the glue line

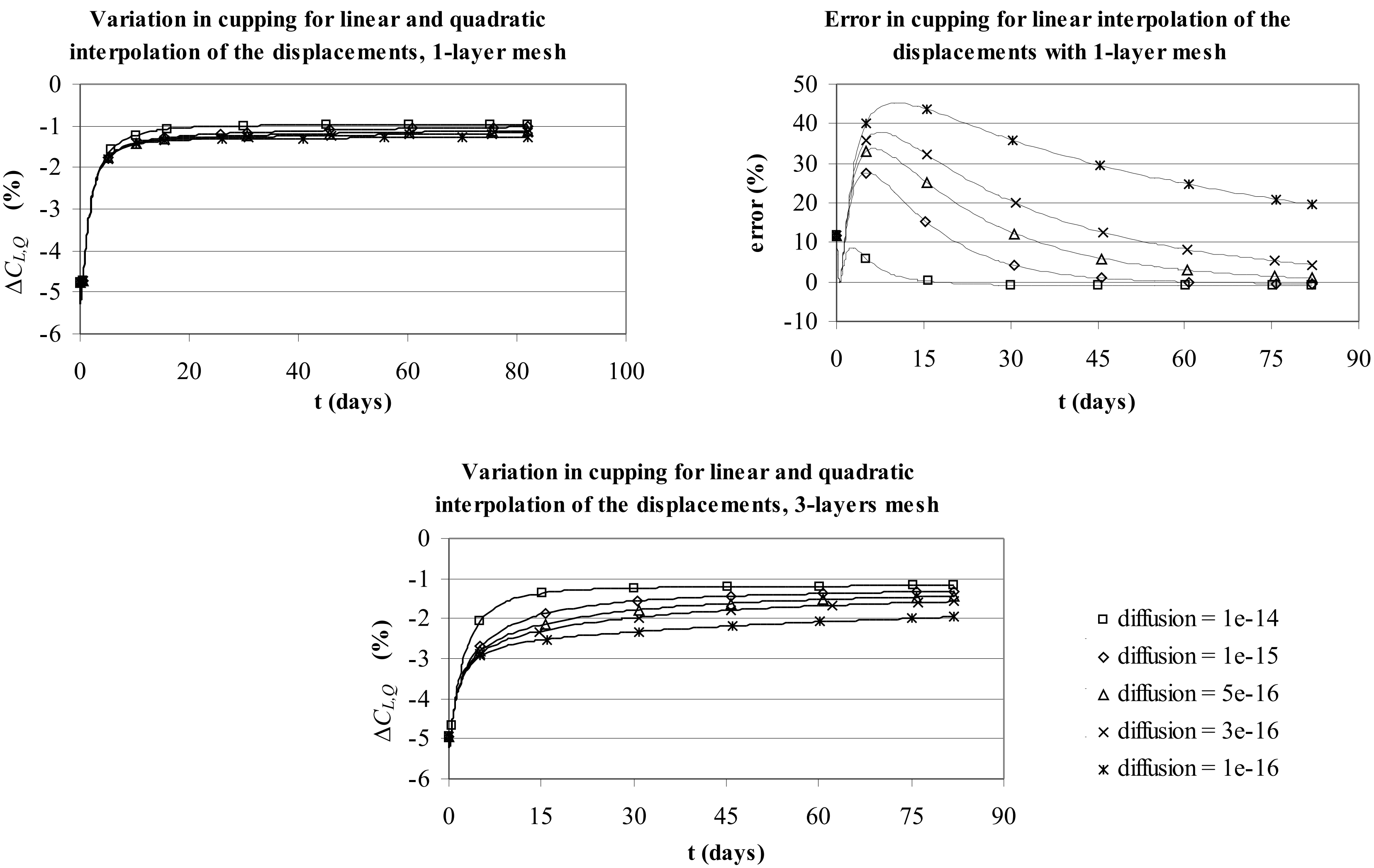


Figure 4 Effects of the degree of interpolation of the displacements on the cupping.

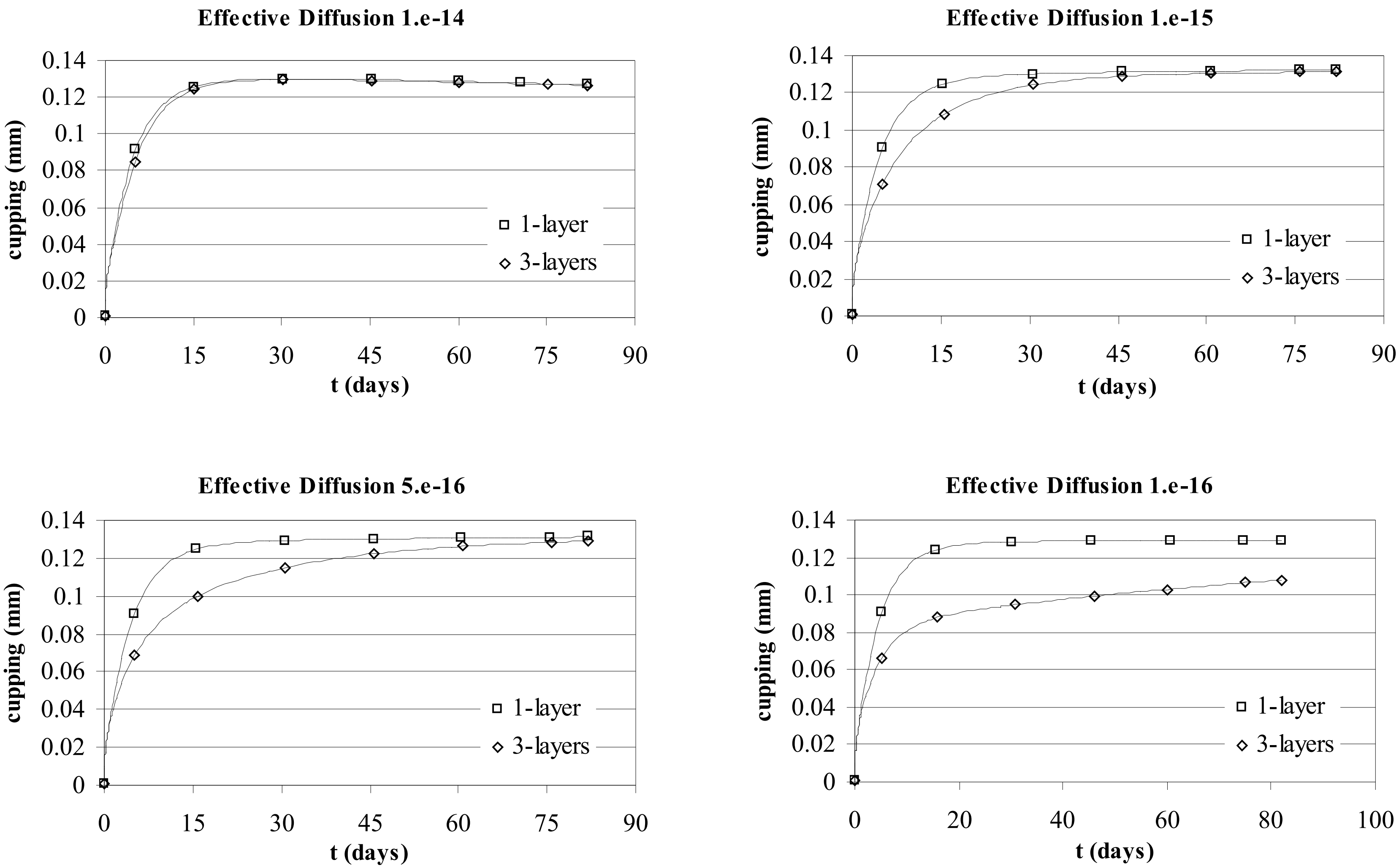


Figure 5 Effect of the number of layers of elements for diminishing effective diffusion in the glue line

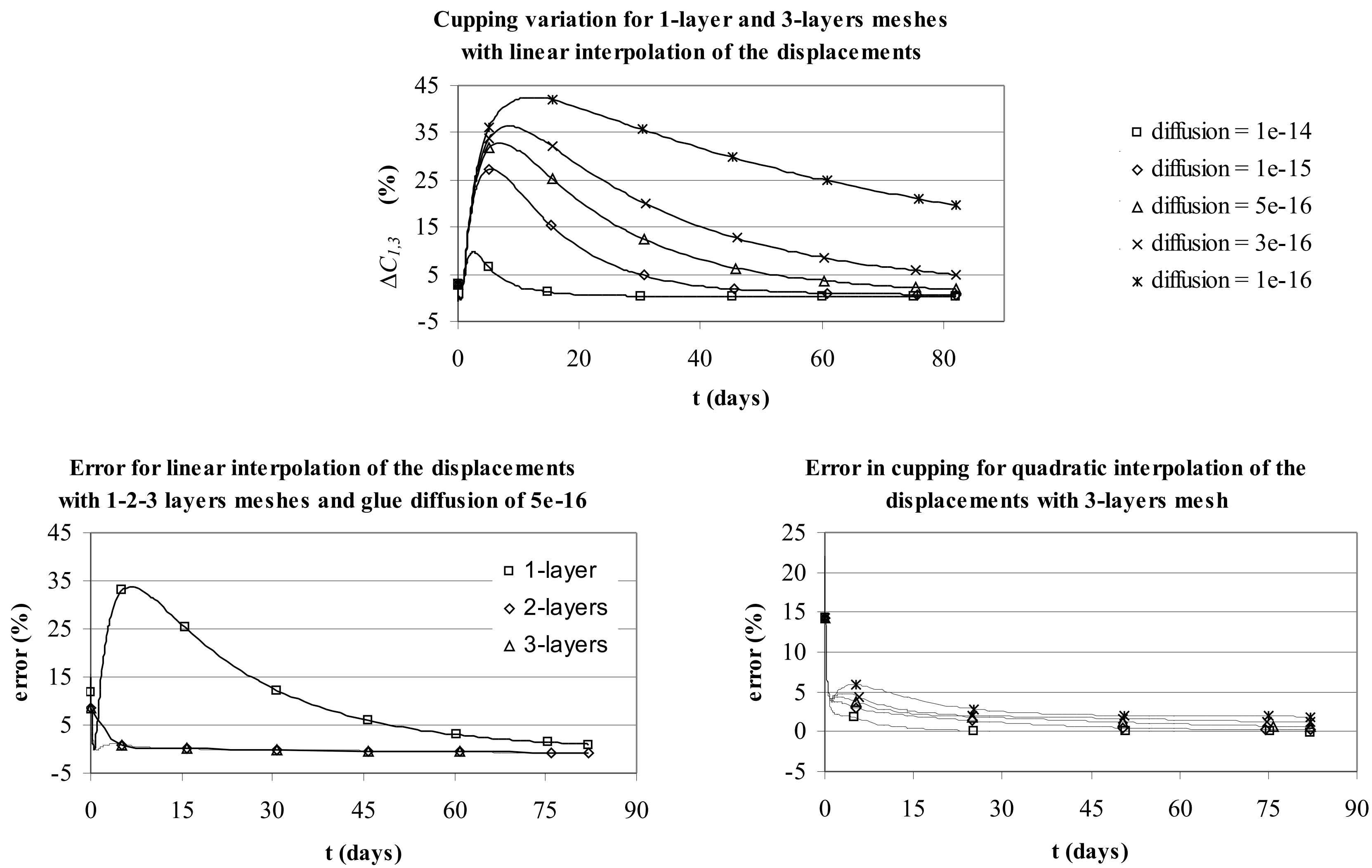
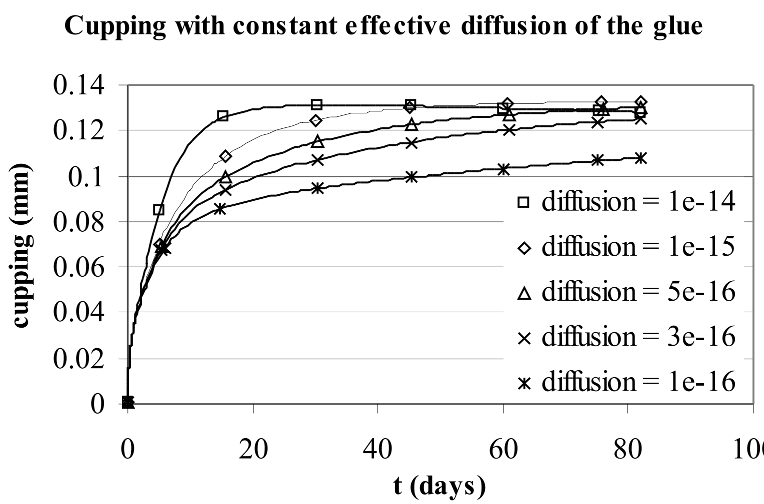
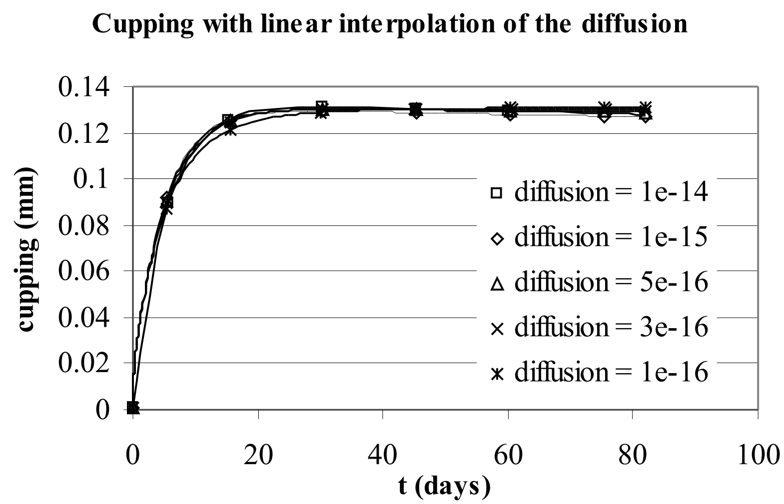
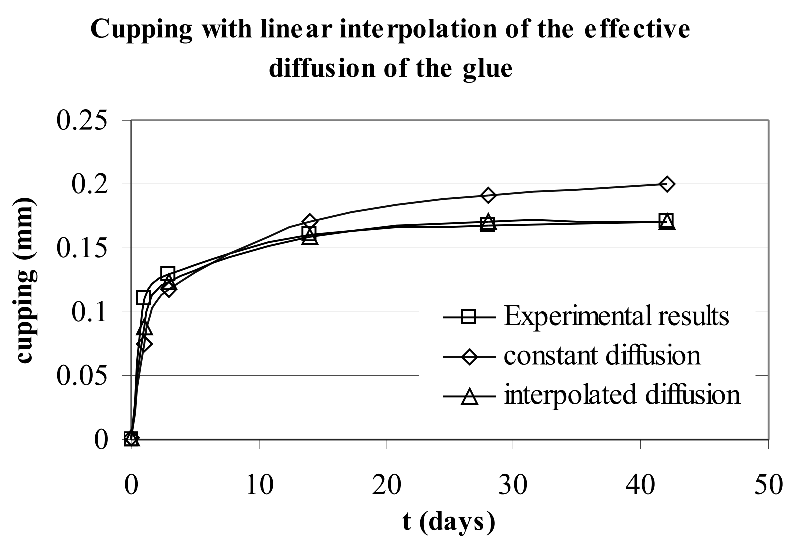
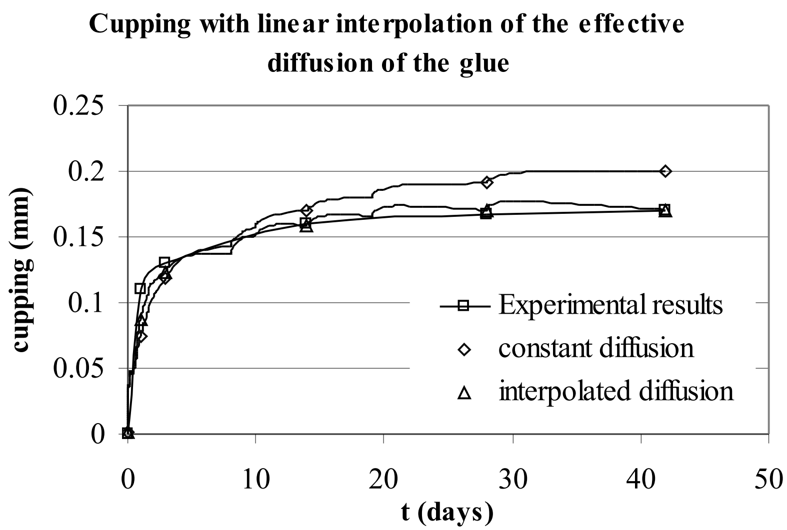


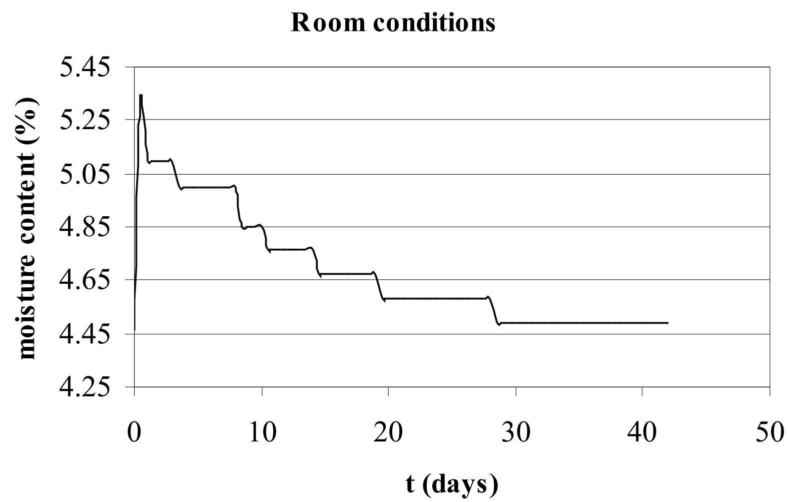
Figure 6 Relative error based on the cupping for 1-layer to 3-layers mesh with various effective diffusion of the glue.


Cupping with constant effective diffusion of the glue
cupping (mm)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0
20
40
60
80
100
t (days)
diffusion = 1e-14
diffusion = 1e-15
diffusion = 5e-16
diffusion = 3e-16
diffusion = 1e-16


Cupping with linear interpolation of the diffusion
cupping (mm)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0
20
40
60
80
100
t (days)
diffusion = 1e-14
diffusion = 1e-15
diffusion = 5e-16
diffusion = 3e-16
diffusion = 1e-16


Cupping with linear interpolation of the effective diffusion of the glue
cupping (mm)
0
0.05
0.1
0.15
0.2
0.25
0
10
20
30
40
50
t (days)
Experimental results
constant diffusion
interpolated diffusion


Cupping with linear interpolation of the effective diffusion of the glue
0.25
0.2
0.15
0.1
0.05
0
cupping (mm)
0
10
20
30
40
50
t (days)
Experimental results
constant diffusion
interpolated diffusion


Room conditions
5.45
5.25
5.05
4.85
4.65
4.45
4.25
moisture content (%)
0
10
20
30
40
50
t (days)