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Applicability of classical isotropic fracture mechanics specimens to wood crack propagation studies

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Different specimen types (double cantilever beam: DCB, compact tension: CT, single edge notched in tension: SENT) have been numerically studied (with special finite elements) for ten wood species and for cracks situated in a material plane of symmetry (the crack is denoted xy with x the direction of the normal to the crack plane and y the direction of propagation). Single-edge notched specimens used in RL, TL directions appear to be insensitive to the orthotropic properties of the material. So, calibration known for isotropic materials can be used in stress intensity factor calculations. In transverse directions (LT, LR), such specimens could be used with a double height. Small differences, but depending on the species tested, have also been obtained in TR and RT directions. Calibration of CT specimens of an isotropic material cannot be used even in TL and RL directions. A calibration acceptable for all the studied species is proposed. A different calibration is also given in the TR direction. It is suggested, however, not to use specimens in the RT direction without a calibration specific for the wood species tested. DCB specimens could be used in RL, TL and TR directions with a calibration which is a function of elastic moduli. This calibration is obtained from analytical calculations. In other directions, these specimens do not offer any experimental interest.

1. INTRODUCTION

For some years, the characterization of composite materials or wood in terms of resistance to crack propagation has widely developed. However, special care is required in applying specimens computed for isotropic materials to orthotropic materials, especially when the orthotropy ratio E_{maxi}/E_{mini} is high, as for wood (E_{maxi} and E_{mini} are the extreme values of the orthotropic Young's modulus).

For an infinite plate containing a crack with a length $2a$ along a material axis, Sih *et al.* [1] have shown that stress intensity factors (SIF) K_m ($m = $ I; II; III) are independent of the elastic constants of the material. In cases of finite-sized specimens including a crack with a length a (the only ones deserving any interest in practice) these factors are given in the form $K_m = \alpha\sigma_m(\pi a)^{1/2}$, σ_m being the applied stress at infinity. For isotropic materials, α only depends on the specimen geometry while, for orthotropic materials, it also depends on elastic constants. Results are given in the literature for some particular cases of elastic constants [2–5].

Walsh [3] has studied more precisely double cantilever beam (DCB) specimens and single edge notched specimens in bending or in tension (SENB or SENT) (Fig. 1). He showed that the former were very sensitive to orthotropy, contrary to the latter. However, these studies only concerned Eucalyptus.

Hunt and Croager [6] investigated Scots Pine using compact tension (CT) (Fig. 1), DCB and SENB test specimens for a unique crack length. For the three specimens, he noticed differences between the isotropic and orthotropic results.

Thus, a reduction can be observed in differences due to reduction of the ratio $2H/W$, where $2H$ and W are respectively the height (or the dimension perpendicular to the crack) and the nominal length (or the dimension parallel to the crack) of the specimen.

The results obtained by Hunt and Croager [6] are shown in Fig. 2, where the stress intensity factor K_I is given according to the ratio $2H/W$. This ratio appears to be most important.

The aim of this paper is therefore to assess numerically the sensitivity of the common tensile test specimens (DCB, CT, SENT) for different wood species and crack lengths. However, the CT test specimen will be more precisely detailed.

2. DOUBLE CANTILEVER BEAM SPECIMEN

Valentin and Lahna [7] have observed, after calculations with the finite element method (FEM), the great sensitivity to orthotropy of a DCB specimen in plane stress in RL and TL directions.

The ratio $R_m = K_{m\,ortho}/K_{m\,iso}$ ($m = $ I; II) could vary, according to the wood species, from 0.3 to 0.5 if $m = $ I and

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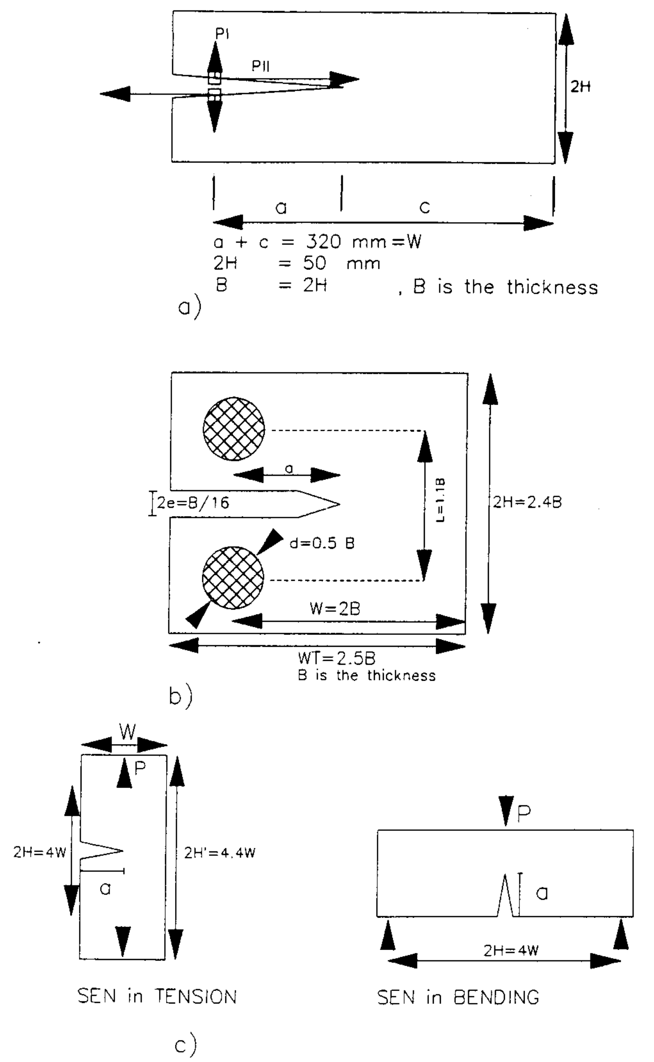

Fig. 1. Common test specimens: (a) double cantilever beam specimen, (b) compact tension specimen, (c) single-edge notched specimen.

from 0.6 to 0.7 if $m = \text{II}$. The study concerned Douglas Fir, Scots Pine and Eucalyptus. The results in mode I are shown in Fig. 3 with other species whose characteristics are given in Table 1.

The analysis of the calculated values shows that, in fact, the sensitivity to orthotropy is particularly high when the ratio E_1/E_2 itself is high. Here E_1 and E_2 actually represent Young's modulus respectively in the crack propagation direction and in the perpendicular direction.

An extension to orthotropic materials of the theory of the homogeneous beam on an elastic foundation model, developed by Kanninen [11], has been realized by Valentin and Lahna [7]. They showed that

$$K_{\mathrm{I\,ortho}}(a) = \frac{P}{B}\left\{\frac{12}{\lambda^2 H^3}\left[\lambda a\left(\frac{\sinh^2(\lambda c)+\sin^2(\lambda c)}{\sinh^2(\lambda c)-\sin^2(\lambda c)}\right) + \left(\frac{\sinh(\lambda c)\cosh(\lambda c)-\sin(\lambda c)\cos(\lambda c)}{\sinh^2(\lambda c)-\sin^2(\lambda c)}\right)\right]^2 + \frac{6\beta}{5H}\right\}^{1/2} \times \left\{\left(\frac{\alpha_{22}}{2\alpha_{11}}\right)\left[\left(\frac{\alpha_{22}}{\alpha_{11}}\right)^{1/2} + \frac{2\alpha_{12}+\alpha_{66}}{2\alpha_{11}}\right]\right\}^{-1/4} \quad (1)$$

where $\lambda = (1/H)(6\alpha_{11}/\alpha_{22})^{1/4}$, $\beta = \alpha_{66}/\alpha_{11}$; P, B, H and c are defined in Fig. 1a, and α_{ij} are the elastic compliances such as $\varepsilon_{ij} = \alpha_{ij}\sigma_{ij}$.

The results given by this relation confirm the numerical results obtained with the FEM. Equation 1 leads to the isotropic model of Kanninen [11] if the term $6\beta/5H$ induced by the shear stress is ignored. Thus we get

$$K_{\mathrm{I\,iso}}(a) = \frac{Pa[3.464 - 2.213(H/a)]}{BH^{3/2}} \quad (2)$$

Nevertheless, the use of Equation 1 proves to be tricky. It can be approached by a more simple form to obtain

$$K_{\mathrm{I\,ortho}}(a) = \frac{Pa}{BH^{1/2}}\left\{12\left(n+\frac{a}{H}\right)^2\left[1+\left(\frac{nH}{c}\right)^{9/2}\right]+\frac{6\beta}{5}\right\}^{1/2} \left[3^{1/4} n\left(6^{1/2}n^2+\frac{\beta}{2}+\tau\right)^{1/4}\right]^{-1} \quad (3)$$

where $n = [\alpha_{22}/(6\alpha_{11})]^{1/4} = 1/(\lambda H)$, $\tau = \alpha_{12}/\alpha_{11}$ and α_{ij}, λ, β have been defined above.

We can notice that, if s_1 and s_2 are the positive imaginary parts of the roots of the characteristic equation

$$\alpha_{11}s^4 + (2\alpha_{12}+\alpha_{66})s^2 + \alpha_{22} = 0$$

Table 1 Mechanical characteristics

Wood species	H (%)	d ($\mathrm{g\,cm^{-3}}$)	E_L (MPa)	E_R (MPa)	E_T (MPa)	G_{LR} (MPa)	G_{LT} (MPa)	G_{TR} (MPa)	ν_{LR}	ν_{LT}	ν_{RT}
1. Maritime Pine [8]	11.0	0.60	11370	944	703	1350	1615	200	0.320	0.530	0.580
2. Pine [10]	9.7	0.54	16600	1120	580	1780	680	70	0.465	0.449	0.605
3. Fir [10]	13.1	0.31	8020	816	304	558	461	48	0.341	0.331	0.669
4. Standard softwood [10]	12.0	0.45	13100	1000	636	862	745	83.6	0.380	0.420	0.490
5. Standard hardwood [10]	12.0	0.65	14400	1810	1030	1260	971	366	0.390	0.460	0.680
6. Balsa wood [10]	9.0	0.10	2490	120	40	130	90	10	0.309	0.535	0.714
7. Humbertia [10]	9.0	1.28	24000	5630	5110	2980	2800	1840	0.373	0.436	0.503
8. Scots Pine [9]	11.5	–	17000	1080	1070	1450	1360	110	0.370	0.470	0.790
9. Douglas Fir [9]	9.5	0.48	16690	1320	920	1200	–	–	0.367	0.384	0.594
10. Eucalyptus [9]			19240	961	–	1014	–	–	0.500	–	–

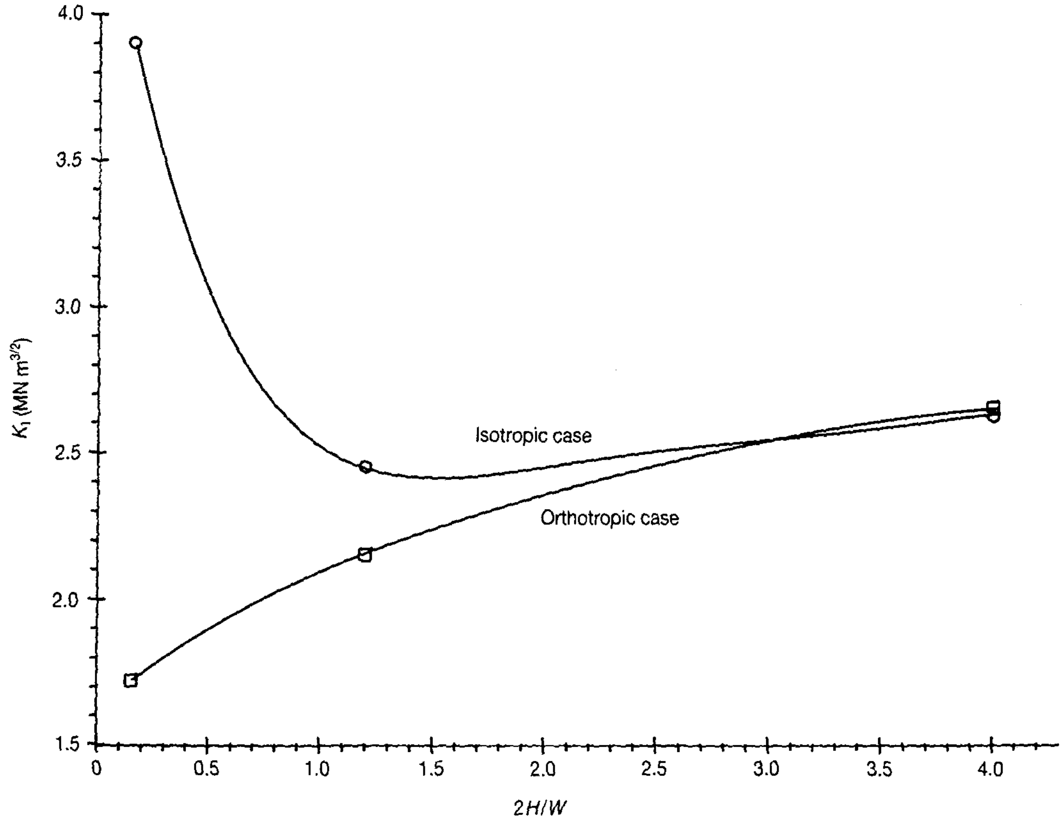

Fig. 2. Interpolation of Hunt's results with both Scots Pine and isotropic solution.

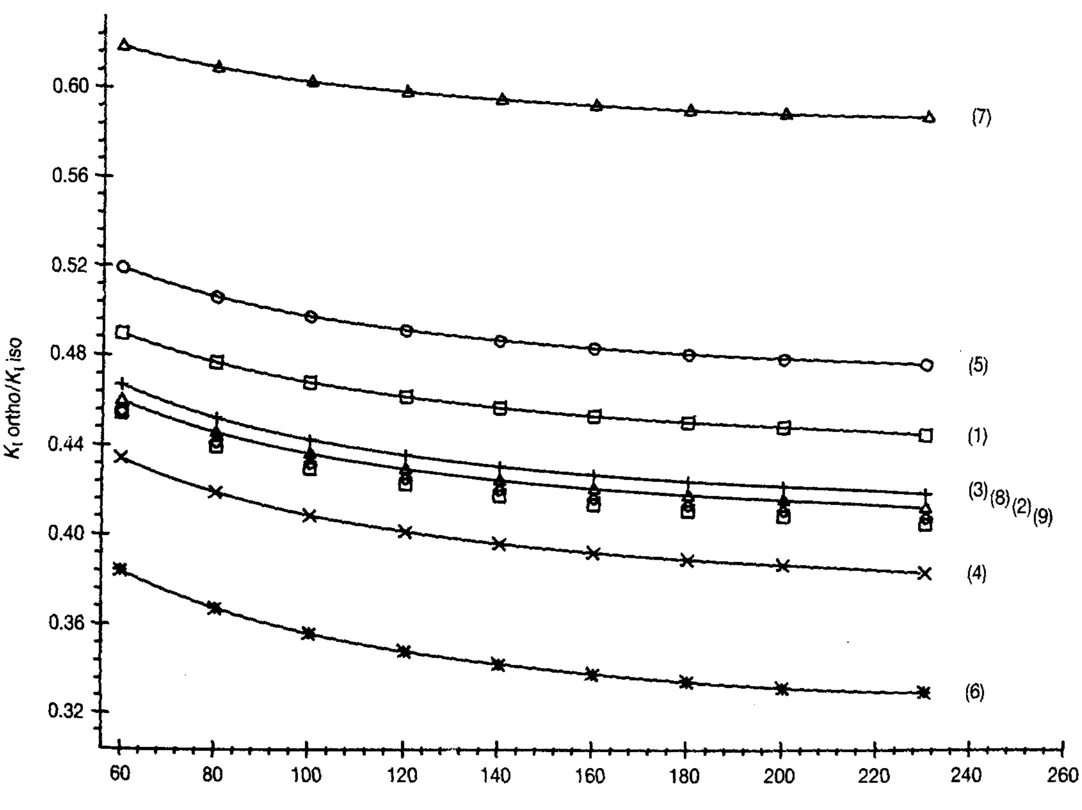

Fig. 3. Orthotropic effects on $K_{I ortho}/K_{I iso}$ versus crack length.

Table 2 Comparison of $K_{\mathrm{Iortho}}(a)$ from Equations 1 and 3 for the different species of Table 1, and the relative differences (%) from Equation 1

Wood species	Equation	Crack length, a (mm)								
		60	80	100	120	140	160	180	200	230
1. Maritime Pine [8] RL	1	10.662	12.946	15.250	17.565	19.889	22.218	24.557	26.915	30.402
	3	10.662 (0.006%)	12.947 (0.009%)	15.252 (0.012%)	17.569 (0.018%)	19.894 (0.029%)	22.228 (0.046%)	24.571 (0.057%)	26.930 (0.057%)	30.573 (0.562%)
TL	1	10.258	12.433	14.622	16.820	19.023	21.233	23.456	25.690	29.004
	3	10.259 (0.008%)	12.434 (0.011%)	14.624 (0.016%)	16.824 (0.025%)	19.031 (0.039%)	21.245 (0.055%)	23.469 (0.055%)	25.714 (0.092%)	29.213 (0.719%)
2. Pine [10] RL	1	9.911	11.981	14.070	16.170	18.277	20.391	22.519	24.658	27.831
	3	9.911 (0.008%)	11.982 (0.011%)	14.072 (0.016%)	16.174 (0.024%)	18.284 (0.039%)	20.402 (0.055%)	22.531 (0.055%)	24.680 (0.089)	28.029 (0.715%)
TL	1	7.647	9.037	10.453	11.887	13.334	14.795	16.264	17.717	20.235
	3	7.648 (0.014%)	9.039 (0.021%)	10.457 (0.032%)	11.892 (0.046%)	13.341 (0.055%)	14.803 (0.053%)	16.280 (0.100%)	17.787 (0.396%)	20.232 (−1.547%)
3. Fir [10] RL	1	10.165	12.274	14.412	16.568	18.736	20.913	23.100	25.307	28.583
	3	10.166 (0.005%)	12.274 (0.007%)	14.413 (0.010%)	16.572 (0.016%)	18.741 (0.025%)	20.922 (0.042%)	23.113 (0.057%)	25.321 (0.053%)	28.720 (0.478%)
TL	1	8.025	9.550	11.096	12.655	14.225	15.809	17.400	18.974	21.638
	3	8.027 (0.014%)	9.552 (0.020%)	11.099 (0.030%)	12.661 (0.044%)	14.233 (0.056%)	15.817 (0.054%)	17.416 (0.089%)	19.043 (0.361%)	21.670 (0.147%)
4. Standard softwood [10] RL	1	9.453	11.363	13.301	15.255	17.220	19.194	21.182	23.184	26.147
	3	9.454 (0.007%)	11.364 (0.010%)	13.303 (0.014%)	15.258 (0.022%)	17.226 (0.035%)	19.204 (0.052%)	21.193 (0.055%)	23.202 (0.075%)	26.325 (0.680%)
TL	1	8.431	10.058	11.709	13.375	15.052	16.741	18.444	20.139	22.799
	3	8.432 (0.011%)	10.060 (0.016%)	11.712 (0.023%)	13.380 (0.036%)	15.060 (0.051%)	16.751 (0.056%)	18.455 (0.060%)	20.184 (0.222%)	22.933 (0.586%)
5. Standard hardwood [10] RL	1	11.300	13.739	16.208	18.695	21.193	23.700	26.213	28.743	32.546
	3	11.301 (0.003%)	13.740 (0.005%)	16.209 (0.007%)	18.697 (0.011%)	21.197 (0.018%)	23.707 (0.030%)	26.226 (0.050%)	28.759 (0.056%)	32.630 (0.258%)
TL	1	9.655	11.617	13.607	15.614	17.632	19.659	21.699	23.756	26.796
	3	9.656 (0.006%)	11.618 (0.009%)	13.609 (0.013%)	15.617 (0.021%)	17.638 (0.033%)	19.669 (0.050%)	21.711 (0.056%)	23.772 (0.067%)	26.969 (0.646%)
6. Balsa wood [10] RL	1	8.359	9.960	11.587	13.229	14.884	16.551	18.232	19.907	22.522
	3	8.359 (0.010%)	9.961 (0.015%)	11.589 (0.023%)	13.234 (0.035%)	14.892 (0.050%)	16.560 (0.056%)	18.242 (0.058%)	19.949 (0.212)	22.658 (0.607%)
TL	1	6.417	7.502	8.607	9.726	10.857	11.989	13.116	14.320	17.500
	3	6.420 (00.33%)	7.505 (0.045%)	8.611 (0.059%)	9.731 (0.054%)	10.864 (0.069%)	12.012 (0.188%)	13.183 (0.505%)	14.405 (0.593%)	17.200 (−1.710%)
7. Humbertia [10] RL	1	13.458	16.533	19.639	22.763	25.899	29.042	32.190	35.345	40.132
	3	13.458 (0.002%)	16.534 (0.003%)	19.640 (0.004%)	22.765 (0.006%)	25.901 (0.010%)	29.047 (0.016%)	32.200 (0.030%)	35.364 (0.052%)	40.159 (0.068%)
TL	1	13.315	16.343	19.402	22.480	25.569	28.667	31.770	34.880	39.599
	3	13.315 (0.002%)	16.343 (0.003%)	19.403 (0.004%)	22.481 (0.006%)	25.572 (0.010%)	28.672 (0.017%)	31.780 (0.030%)	34.899 (0.053%)	39.628 (0.071%)
8. Scots Pine [9] RL	1	10.010	12.096	14.206	16.331	18.466	20.608	22.763	24.934	28.144
	3	10.011 (0.006%)	12.097 (0.008%)	14.208 (0.012%)	16.335 (0.019%)	18.472 (0.031%)	20.618 (0.057%)	22.775 (0.057%)	24.949 (0.060%)	28.312 (0.599%)
9. Douglas Fir [9] RL	1	9.890	11.926	13.990	16.070	18.161	20.260	22.372	24.501	27.650
	3	9.891 (0.006%)	11.927 (0.008%)	13.991 (012%)	16.073 (0.019%)	18.166 (0.030%)	20.270 (0.047%)	22.384 (0.057%)	24.516 (0.059%)	27.813 (0.591%)

Equation 3 becomes slightly more simple and we get

$$K_{\mathrm{I\,ortho}}(a)=P\left[\left[12\left[\left(\frac{s_1s_2}{6^{1/2}}\right)^{1/2}+\frac{a}{H}\right]^2\right.\right.$$
$$\times\left\{1+\left[\left(\frac{s_1s_2}{6^{1/2}}\right)^{1/2}\frac{H}{c}\right]^{9/2}\right\}^2+\frac{6}{5}(s_1^2+s_2^2-2\tau)\Bigg]\Bigg]^{1/2}$$
$$\times\left\{BH^{1/2}\left[s_1s_2\left(\frac{s_1+s_2}{2}\right)\right]^{1/2}\right\}^{-1} \quad (4)$$

For all species listed in Table 1, the relative differences between the complete Equation 1 and the approximate Equations 3 or 4 are still lower (in absolute value) than 2% for crack lengths a between 50 and 230 mm, i.e. $2\leq a/H\leq 9$. Results are shown in Table 2 from which we can deduce that, in general, differences are definitely lower than 0.5%.

Using an identical principle, Komatsu *et al.* [12] have studied a DCB specimen with an adhesive layer of finite thickness. An expression for SIF K_I taking into account the characteristics of the adhesive layer is obtained. If the correction term $(nH/c)^{9/2}$ is overlooked in the approximate. Equation 3, Komatsu *et al.*'s formula is to be found again when the adhesive layer thickness gets near to zero.

Every DCB test specimen following the conditions expressed above, in which Equation 1 has been stated and verified, i.e. $W/H\approx 12$ and $0.2\leq a/W\leq 0.7$, can be studied with any of Equations 1, 3 and 4.

3. COMPACT TENSION SPECIMEN

3.1 Numerical calculation of K_I

As it is very convenient to manufacture and utilize and it requires very little material, this test specimen is more and more widely used for orthotropic materials. Wright and Fonselius [13] and Barrett and Foschi [14], among others, have used it on wood. Its sizes, standardized by the ASTM [15] for isotropic metallic materials, are already described in Fig. 1b.

The test specimen is studied below, assuming that the material is in a state of plane strain. In a common practice nowadays, the special elements of Henshell and Shaw [16] and Barsoum [17] are used at the crack tip, as shown in Fig. 4b. Crack length variation is induced by a translation of the hatched area in Fig. 4a.

Three methods used here are now described.

3.1.1 *Extrapolating crack tip displacements (M1)*

K_I is computed from opening displacements of the first four nodes behind the crack tip. In the isotropic case we have

$$K_{\mathrm{I\,iso}}^{(i)}=\frac{2E}{4(1-\nu^2)}\left(\frac{2\pi}{r_i}\right)^{1/2}u_y^{(i)} \qquad i=1,2,3,4 \quad (5)$$

where E, ν are respectively Young's modulus and Poisson's ratio of the material, while r_i is the distance between the tip and the node i and $u_y^{(i)}=u_y^{(i)}(\theta=+\pi)$ is the opening displacement of node i. A linear extrapolation to the crack tip $(r=0)$ gives $K_I(a)$.

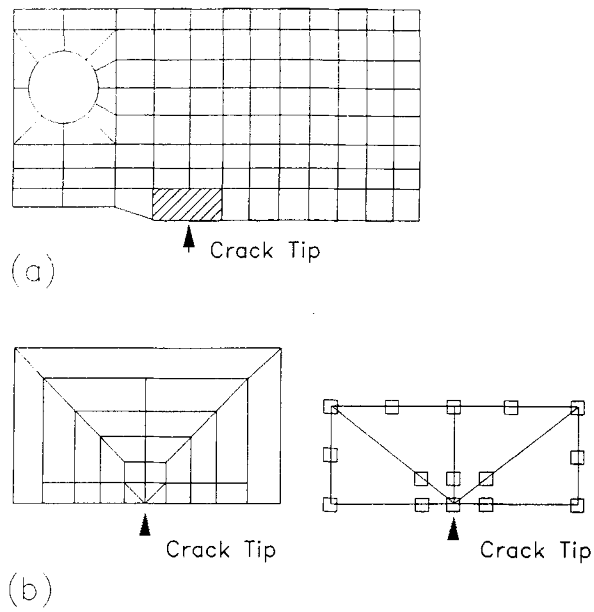

Fig. 4. Proposed mesh: (a) mesh of the half-specimen, (b) enlargement of the hatched zone.

In the orthotropic case, there is

$$K_{\mathrm{I\,ortho}}^{(i)}=\left(\frac{\pi}{2r_i}\right)^{1/2}[F(s_1,s_2)]^{-1}u_y^{(i)} \qquad i=1,2,3,4 \quad (5')$$

where

$$[F(s_1,s_2)]^{-1}$$
$$=\mathrm{Re}\,\{(s_1-s_2)^{-1}[s_1q_2(\cos\theta+s_2\sin\theta)^{1/2}$$
$$-s_2q_1(\cos\theta+s_1\sin\theta)^{1/2}]\}$$

s_1, s_2, α_{ij} have been defined above by Equations 1, 3 and 4, and $q_i=(1/s_i)(\alpha_{12}s_i^2+\alpha_{22})$ where $i=1, 2$. Note that $\mathrm{Re}(z)$ is the real part of a complex number z.

In the case of plane strain in plane (i,j) where $i,j=1,2$, we have

$$\alpha_{ij}=S_{ij}-(S_{i3}S_{j3})/S_{33} \qquad i=1,2,6$$

where S_{ij} are the coefficients of the compliance matrix of the orthotropic material such as $\{\varepsilon\}=[S]\{\sigma\}$ (Hooke's law).

3.1.2 *Crack opening displacement method (M2)*

This is the most direct because it uses only special element nodes behind the crack tip [18]. In the isotropic case, there is

$$K_{\mathrm{I\,iso}}(a)=\left(\frac{2\pi}{l_c}\right)^{1/2}\frac{E}{4(1-\nu^2)}(4u_y^{(1)}-u_y^{(2)}) \quad (6)$$

where (1) and (2) refer respectively to the first and second nodes behind the crack tip and l_c is the distance between node (2) and the crack tip (or the singular element's side length).

In the orthotropic case there is

$$K_{\mathrm{I\,ortho}}(a)=\left(\frac{\pi}{2l_c}\right)^{1/2}[F(s_1,s_2)]^{-1}(4u_y^{(1)}-u_y^{(2)}) \quad (6')$$

Table 3 Comparison of $F(a/W)=K_IBW^{1/2}/P$ calculated by the three methods M1, M2 and M3, with the relative differences (%) from Equation 1

Method	Crack length, a/W				
	0.3	0.4	0.5	0.6	0.7
Isotropic solution (ASTM [15])	5.62	7.28	9.66	13.65	21.55
Method M1	5.60 (−0.40%)	7.22 (−0.80%)	9.56 (−1.00%)	13.53 (−0.90%)	21.40 (−0.70%)
Method M2	5.57 (−0.90%)	7.18 (−1.30%)	9.50 (−1.60%)	13.44 (−1.60%)	21.22 (−1.50%)
Method M3	5.62 (0.00%)	7.28 (0.00%)	9.67 (0.20%)	13.72 (−0.50%)	21.77 (1.00%)

3.1.3 *The virtual propagation method (M3)*

This consists in making the crack length vary by a quantity δa and then calculating the strain energy release rate using the following form:

$$G_I = -\frac{P^2}{2}\left(\frac{\delta C(a)}{\delta a}\right) = \frac{\delta U}{\delta a} \tag{7}$$

where P is the applied load on the structure, $C(a)$ is the compliance of the structure having a crack length a and δU is the energy necessary for the crack propagation from a to $a+\delta a$. For an isotropic material, K_I is such that

$$K_I^2 = \frac{E}{1-\nu^2} G_I \tag{7'}$$

For an orthotropic material, K_I is such that

$$K_I^2 = \left\{\left(\frac{\alpha_{11}\alpha_{22}}{2}\right)^{1/2}\left[\left(\frac{\alpha_{11}}{\alpha_{22}}\right)^{1/2} + \frac{2\alpha_{12}+\alpha_{66}}{2\alpha_{11}}\right]^{1/2}\right\}^{-1} G_I \tag{7''}$$

In the following, $F(a/W)$ will be called the normalized quantity $K_IBW^{1/2}/P$. The mesh and the three methods above have been validated on an isotropic material which is characterized by $v=0.3$ and $E=210\,000$ MPa. The results are listed in Table 3 where they are compared to the calibration formula given by ASTM [15].

The mean relative differences observed are −0.75% with the extrapolating method M1 and 0.35% with method M3. The COD method M2 gives the worst results with an average difference of −1.3%.

Only the first method, reliable and easier to implement, will be used from now on.

3.2 Stress intensity factors of wood test specimens

The six basic configurations RL–TL; RT–TR; LR–LT, according to the standard notation of Johnson [19], in the plane strain state have been studied with the materials listed above in Table 1.

These characteristics are derived from Cariou [8] and Lahna's [9] work and from a compilation realized by Guitard [10]. The results are gathered in Tables 4 to 8

Table 4 Values of $F(a/W)_{ortho}$ for the different species of Table 1, with the relative differences (%) from the isotropic solution [15]

Method		Crack length, a/W				
		0.3	0.4	0.5	0.6	0.7
Isotropic solution (ASTM [15])		5.62	7.28	9.66	13.65	21.55
1. Maritime Pine [18]	RL	4.92 (−12.40%)	6.58 (−9.50%)	9.08 (−5.90%)	13.26 (−2.80%)	21.26 (−1.30%)
	TL	4.91 (−12.50%)	6.58 (−9.50%)	9.12 (−5.50%)	13.35 (−2.10%)	21.43 (−0.50%)
2. Pine [10]	RL	4.91 (−12.50%)	6.57 (−9.60%)	9.08 (−5.90%)	13.27 (−2.70%)	21.28 (−1.20%)
	TL	4.90 (−12.70%)	6.53 (−10.20%)	8.98 (−6.90%)	13.07 (−4.20%)	20.90 (−2.90%)
3. Fir [10]	RL	5.00 (−11.00%)	6.61 (−9.00%)	9.03 (−6.40%)	13.09 (−4.10%)	20.91 (−2.90%)
	TL	4.91 (−12.50%)	6.55 (−9.90%)	9.03 (−6.40%)	13.18 (−3.40%)	21.09 (−2.10%)
4. Standard softwood [10]	RL	4.94 (−11.90%)	6.57 (−9.60%)	9.02 (−6.50%)	13.11 (−3.90%)	20.97 (−2.60%)
	TL	4.91 (−12.50%)	6.55 (−9.90%)	9.02 (−6.50%)	13.14 (−3.70%)	21.02 (−2.40%)
5. Standard hardwood [10]	RL	5.03 (−10.30%)	6.65 (−8.60%)	9.06 (−6.10%)	13.11 (−3.90%)	20.95 (−2.70%)
	TL	4.94 (−11.90%)	6.57 (−9.60%)	9.02 (−6.50%)	13.11 (−3.90%)	20.96 (−2.70%)
6. Balsa wood [10]	RL	4.92 (−12.50%)	6.55 (−9.90%)	9.01 (−6.70%)	13.11 (−3.90%)	20.97 (−2.60%)
	TL	4.92 (−12.50%)	6.54 (−10.00%)	9.02 (−6.60%)	13.15 (−3.70%)	21.04 (−2.30%)
7. Humbertia [10]	RL	5.15 (−8.30%)	6.75 (−7.20%)	9.13 (−5.40%)	13.16 (−3.60%)	20.98 (−2.60%)
	TL	5.15 (−8.30%)	6.75 (−7.20%)	9.12 (−5.50%)	13.14 (−3.70%)	20.95 (−2.80%)

Table 5 Values of $F(a/W)_{ortho}$ according to a/W, the maximum relative differences (%) of $F(a/W)_{ortho}$ to $\bar{F}(a/W)_{ortho}$, and the ratio of $\bar{F}(a/W)_{ortho}$ to the isotropic solution [15]

	Crack length, a/W				
	0.3	0.4	0.5	0.6	0.7
$\bar{F}(a/W)$ for RL and TL directions	5.025 (±2.50%)	6.64 (±2.50%)	6.64 (±0.83%)	13.21 (±1.05%)	21.16 (±0.95%)
$\bar{F}(a/W)/F(a/W)$ [15]	0.894	0.912	0.937	0.967	0.982

below, allowing a division of the analysis into three parts according to the selected type of configuration.

The species in Table 1 cover a wide spectrum of densities (0.1–1.28 g cm^{-3}) corresponding to extreme orthotropy ratios E_{maxi}/E_{mini} and going from about 5 for the heaviest material to 60 for the lightest one. Thus, the comments about the CT specimen will be of a general order.

3.2.1 *RL and TL directions*

These directions, corresponding to planes of weakness in wood (longitudinal crack), are the most frequently studied. The calculations are made on species 1 to 7 and the results are listed with the differences from the isotropic solution [15] in Table 4. Both directions RL and TL give quite close values of K_I, but always lower (6% average) than those obtained with an isotropic material.

For each crack length, calculations have been made from extreme values of $F(a/W)_{ortho}$, giving an average value. The maximum relative difference from the values really obtained did not exceed 2.5% in absolute value, as shown in Table 5.

A third-degree polynomial fitting provides an average calibration polynomial ($\bar{F}$ indicates this average value):

$$\bar{F}(a/W)_{ortho} = 250.8(a/W)^3 - 273.3(a/W)^2 + 115.5(a/W) - 11.8 \quad (8)$$

The curve in Fig. 5 shows for RL and TL directions the different values of the following ratio:

$$\bar{R}_I = \frac{\bar{F}(a/W)_{ortho}}{F(a/W)_{iso}}$$

It therefore allows the sensitivity to orthotropy of CT specimens to be measured according to crack length. Thus, we noticed that the orthotropy effect decreases with increasing crack length. This length has a great influence, as $\bar{R}_I$ presents a 10% variation between $a/W = 0.3$ and $a/W = 0.7$.

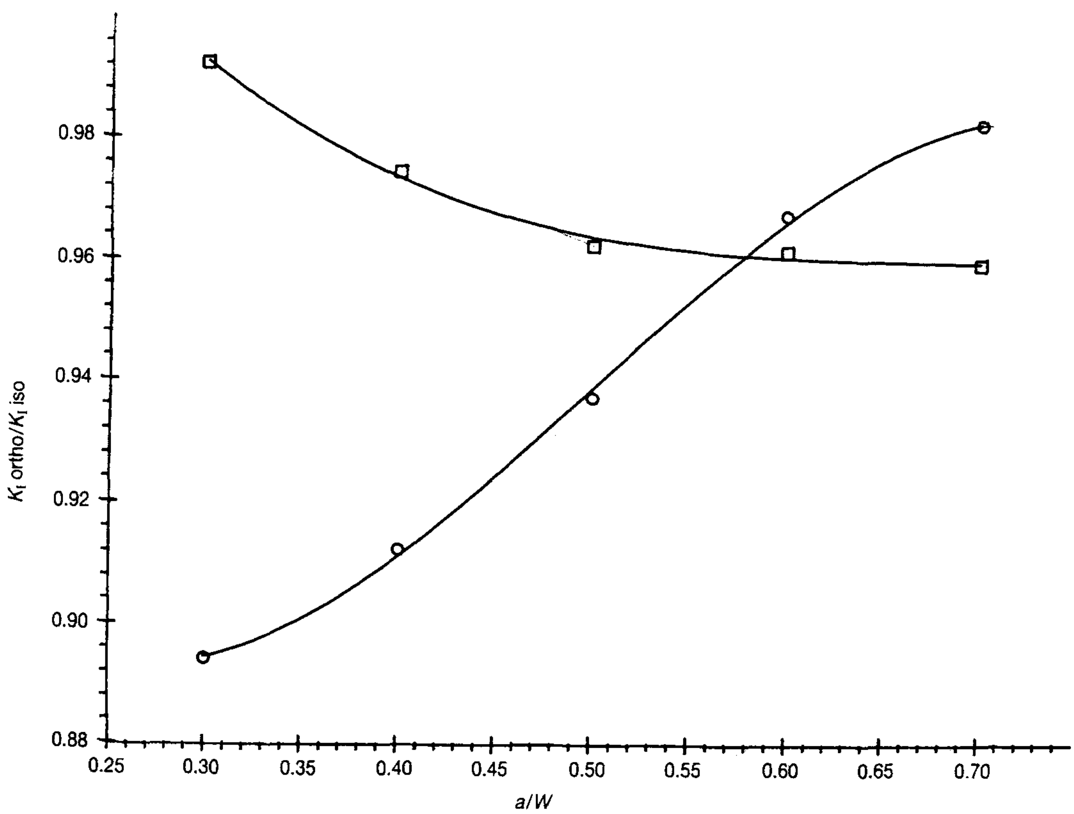

Fig. 5. Sensitivity of CT specimen to orthotropy for both RL-TL (○) and TR (□) directions.

Table 6 Values of $F(a/W)_{ortho}$ for the different species of Table 1, with the relative differences (%) from the isotropic solution [15]

	Crack length, a/W				
	0.3	0.4	0.5	0.6	0.7
Isotropic solution (ASTM [15])	5.62	7.28	9.66	13.65	21.55
1. Maritime Pine [8] TR	5.50 (−2.00%)	7.07 (−2.70%)	9.37 (−2.90%)	13.30 (−2.50%)	21.05 (−2.20%)
RT	5.94 (5.70%)	7.48 (2.70%)	9.69 (0.30%)	13.48 (−1.30%)	21.08 (−2.20%)
2. Pine [10] TR	5.80 (3.30%)	7.21 (−0.80%)	9.32 (−3.40%)	12.94 (−5.10%)	20.16 (−6.40%)
RT	6.68 (18.80%)	8.03 (10.30%)	9.98 (3.30%)	13.27 (−2.80%)	19.87 (−7.80%)
3. Fir [10] TR	5.59 (−0.50%)	7.04 (−3.20%)	9.20 (−4.60%)	12.93 (−5.20%)	20.32 (−5.60%)
RT	6.94 (23.50%)	8.31 (14.10%)	10.25 (6.10%)	13.49 (−1.10%)	20.00 (−7.20%)
4. Standard softwood [10] TR	5.84 (3.90%)	7.26 (−0.10%)	9.39 (−2.70%)	13.04 (−4.40%)	20.33 (−5.60%)
RT	6.48 (15.20%)	7.86 (8.00%)	9.87 (2.20%)	13.30 (−2.60%)	20.22 (−6.20%)
5. Standard hardwood [10] TR	5.31 (−5.40%)	6.92 (−4.80%)	9.28 (−3.80%)	13.30 (−2.50%)	21.16 (−1.80%)
RT	6.17 (9.80%)	7.72 (6.10%)	9.90 (2.50%)	13.63 (−0.20%)	21.19 (−1.70%)

3.2.2 *RT and TR directions*

Species 1 to 5 of Table 1 will be used. Here, the ratio E_1/E_2 defined above in Section 2 is rather close to 1, higher for the TR direction and lower for the RT direction. Thus, the behaviour of CT specimens in both directions studied should be close to that observed for an isotropic material.

Table 6 confirms this for the TR direction. The relative differences from the isotropic solution [15] do not exceed 6.5% in absolute value, that is one-half of the observed differences for RL and TL directions.

We suggest, for the TR direction, an average calibration estimated with extreme values of $F(a/W)_{ortho}$ and obtained from a third-degree fitting. It is considered a satisfactory estimation for all species, to within ±5%, as can be seen in Table 7:

$$\bar{F}(a/W)_{ortho} = 252.9(a/W)^3 - 281.7(a/W)^2 + 119.6(a/W) - 11.8 \quad (9)$$

The curve in Fig. 5 gives, for the TR direction, the sensitivity of CT specimens to orthotropy according to crack length. Crack length has less influence here than in RL or TL directions. The ratio R_I varies by about 4% between $a/W = 0.3$ and $a/W = 0.7$. Unlike the results of RL and TL directions, the maximum difference with the isotropic solution is obtained for the greatest crack lengths.

As far as the RT direction is concerned, our analysis is not so simple. Here, the values obtained for short cracks ($a/W \leq 0.5$) vary considerably according to the materials used. We can think about the influence of the shear modulus whose effect is no longer negligible in this direction in view of E_1 or E_2.

In addition, we noticed that when G_{RT}/E_1 (or E_2) ≈ 10, the differences are biggest between the two directions. This is reversed when G_{RT}/E_1 (or E_2) ≈ 5.

3.2.3 *LR and LT directions*

Materials 1 to 5 of Table 1 were used again. In Table 8, the results show that CT specimens must be handled with care. As a matter of fact, these directions give results which are very different from those obtained for isotropic materials, with many disparities according to the subject species. This fact precludes any reliable use of this test specimen in these directions.

LR and LT directions correspond to planes for which the propagation tends to start in a direction different from that of the initial crack. Pure mode I and K_I values are not relevant and hence of no practical interest.

3.3 Suggested practical relations

For a given crack length, stress intensity factors K_I are lower than those for an isotropic material for RL, TL and

Table 7 Values of $\bar{F}(a/W)_{ortho}$ according to a/W, the maximum relative differences (%) of $F(a/W)_{ortho}$ from $\bar{F}(a/W)_{ortho}$, and the ratio of $\bar{F}(a/W)_{ortho}$ to the isotropic solution [15]

	Crack length, a/W				
	0.3	0.4	0.5	0.6	0.7
$\bar{F}(a/W)$ for TR direction	5.575 (±4.65%)	7.09 (±2.35%)	9.295 (±0.95%)	13.11 (±1.35%)	20.66 (±2.40%)
$\bar{F}(a/W)/F(a/W)$ [15]	0.992	0.974	0.962	0.961	0.959

Table 8 Values of $F(a/W)_{ortho}$ for the different species of Table 1, with the relative differences (%) from the isotropic solution [15]

	Crack length, a/W				
	0.3	0.4	0.5	0.6	0.7
Isotropic solution (ASTM [15])	5.62	7.28	9.66	13.65	21.55
1. Maritime Pine [18] LR	8.94 (59.00%)	10.73 (47.40%)	12.79 (32.50%)	15.95 (16.80%)	22.47 (4.30%)
LT	9.77 (73.80%)	11.79 (62.00%)	13.94 (44.40%)	17.02 (24.70%)	23.35 (8.30%)
2. Pine [10] LR	9.59 (70.60%)	11.51 (58.10%)	13.63 (41.10%)	16.70 (22.30%)	22.95 (6.50%)
LT	10.51 (86.90%)	12.51 (71.80%)	14.71 (52.30%)	17.61 (28.90%)	22.92 (6.30%)
3. Fir [10] LR	8.51 (51.40%)	10.12 (39.00%)	12.12 (25.50%)	15.25 (11.60%)	21.58 (0.10%)
LT	10.81 (92.20%)	12.92 (77.50%)	15.20 (57.30%)	18.14 (32.80%)	20.00 (9.60%)
4. Standard softwood [10] LR	9.12 (62.30%)	10.85 (49.00%)	12.89 (33.40%)	15.93 (16.70%)	22.03 (2.20%)
LT	10.13 (80.20%)	12.07 (65.80%)	14.23 (47.30%)	17.18 (25.80%)	22.86 (6.10%)
5. Standard hardwood [10] LR	7.98 (41.90%)	9.54 (31.00%)	11.53 (19.40%)	14.77 (8.20%)	21.40 (0.70%)
LT	8.98 (59.80%)	10.69 (46.80%)	12.72 (31.60%)	15.77 (15.50%)	21.90 (1.60%)

Table 9 Values of $F(a/W)_{ortho}$ at $a/W = 0.5$ for the different species of Table 1 and isotropic solution [20]

	$F(a/W)_{ortho}$
Isotropic solution [20]	3.53
Computed isotropic value	3.55
1. Maritime Pine [18] RL	3.50
LR	3.47
TR	3.48
RT	3.47
2. Pine [10] RL	3.50
LR	3.46
TR	3.38
RT	3.31
3. Fir [10] RL	3.46
LR	3.47
TR	3.40
RT	3.30
4. Standard softwood [10] RL	3.47
LR	3.37
TR	3.41
RT	3.36
5. Standard hardwood [10] RL	3.47
LR	3.41
TR	3.49
RT	3.48
6. Balsa wood [10] RL	3.47
LR	3.33
TR	3.44
RT	3.37
7. Humbertia [10] RL	3.47
LR	3.44
TR	3.51
RT	3.51

TR directions: For $a/W = 0.5$, our results are very close to those obtained by Hunt and Croager [6].

CT specimens are less sensitive to orthotropy than DCB specimens. The ratio $2H/W$, which is higher for the former, reduces the orthotropy effect.

On the other hand, CT specimens are more sensitive to crack size. As a matter of fact, the variation of ratio R_I between $a/W = 0.3$ and $a/W = 0.7$ corresponds to about 10% for RL and TL directions, while for DCB specimens it has been less than a 5% variation for similar crack size ratios (from $a = 100$ mm to $a = 230$ mm).

For CT specimens, an average calibration estimated with extreme values of $P(a/W)_{ortho}$ and obtained from a fourth-degree fitting can be done, with

$$P(a/W)_{ortho} = F(a/W)_{ortho} \frac{(1 - a/w)^{3/2}}{2 + (a/W)}$$

The following formula is obtained for RL and TL directions:

$$\bar{P}(a/W)_{ortho} = -15.64(a/w)^4 + 32.24(a/W)^3 - 24.19(a/W)^2 + 7.81(a/W) + 0.369 \quad (10)$$

For the TR direction, the formula is

$$\bar{P}(a/W)_{ortho} = -13.46(a/W)^4 + 29.54(a/W)^3 - 22.99(a/W)^2 + 7.05(a/W) + 0.685 \quad (11)$$

Polynomials $P(a/W)_{iso}$[15] and $\bar{P}(a/W)_{ortho}$ of Equations 10 and 11 are shown in Fig. 6.

4. SINGLE-EDGE NOTCHED SPECIMEN

This test specimen has formed the subject of an almost complete numerical study for Eucalyptus by Walsh [3] in pure or simple bending or in pure tension. He has verified that the orthotropy effect for a crack along a material axis became insignificant when the ratio $2H/W$ was higher than 4 for RL and TL cracks or 8 for LR and LT cracks.

For $2H/W = 4$, we get exactly the minimal size of SENB

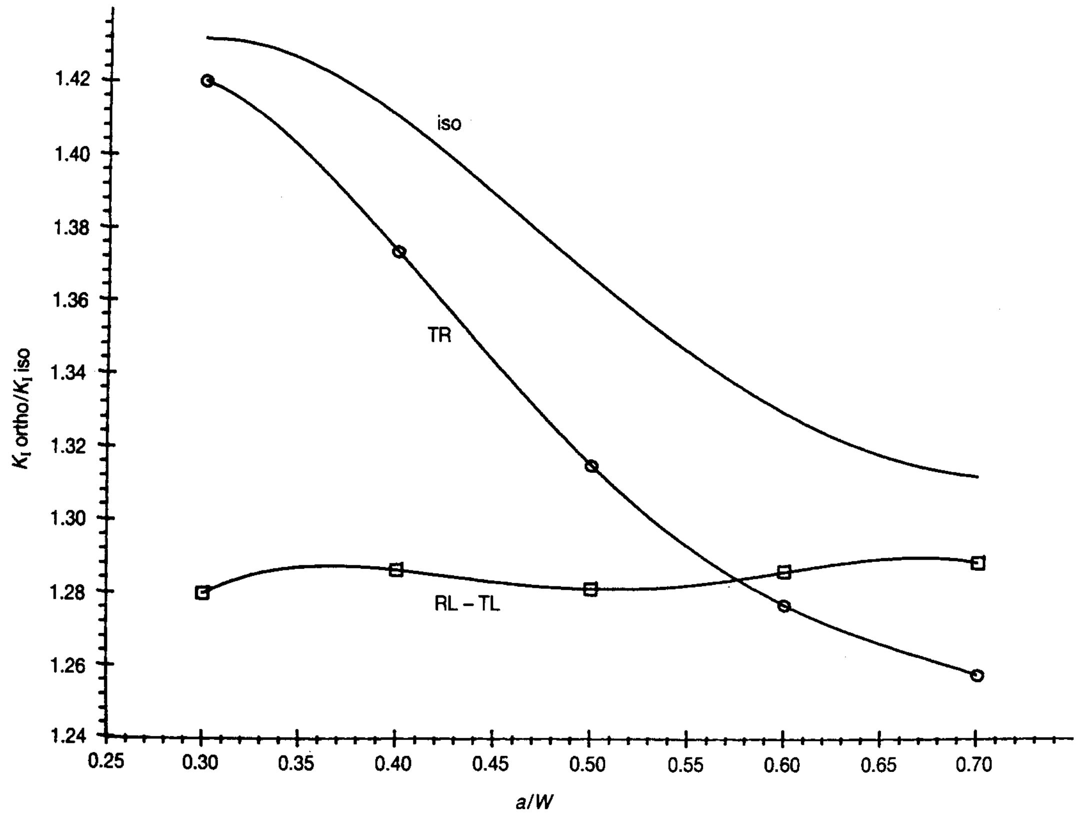

Fig. 6. Polynomials $P(a/W)$ of isotropic solution [15], RL–TL and TR directions versus a/W.

test specimens according to ASTM [15]. They are also the classical sizes of SENT specimens. Walsh's [3] results had been applied only to Eucalyptus in plane stress. Here generalizations have been applied to other species in plane strain. In this study, restriction is made to SEN in tension specimens defined by $2H/W=4$ and $a/W=0.5$.

The first finite-element calculations with the previous isotropic material gave a relative difference of -0.75% when using Equation 12 below as given by Srawley and Gross [20]. This is really satisfactory.

$$F(a/W)=[53.85(a/W)^4-38.48(a/w)^3 + 18.70(a/W)^2-0.41(a/W)+1.99](a/W)^{1/2} \quad (12)$$

The results obtained for RL, LR, TR and RT directions are listed in the Table 9 with the relative differences from the values found for the isotropic material mentioned above. They confirm Walsh's observations for a crack in the RL direction: the isotropic solution is applicable to within 2%. On the other hand, there are always differences for LR, TR and RT directions.

For the LR direction, a reduction of the orthotropy effects can be observed if the test specimen is higher (for example if we take $2H/W=8$ like the one studied by Walsh [3]).

Concerning the TR and RT directions, we can only make a tentative conclusion and anticipate an analogy in the test specimen behaviour in both directions. If the differences between these two directions are smaller than those of CT specimens, the relative differences from the calculated isotropic values are, for the TR direction, of the same order as those observed for CT specimens.

We pointed out that it seems to be useless to consider TL and LT directions for the SENT specimens, since the conclusions are similar to those given with RL and LR directions respectively.

5. CONCLUSIONS

Walsh [3] shows, from his results on Eucalyptus, that SIFs increasingly depend on orthotropy if the ratio $2H/W$ is reduced. We were able to verify the validity of these remarks on a number of wood species. This led to the proposition of an approximated expression for K_I valid for an orthotropic material for DCB test specimens (RL or TL).

If the material orthotropy has less effect on a CT specimen than on a DCB specimen, crack length has, on the other hand, a greater influence on the former.

Two third-degree approximations have been suggested, taking into account the evolution of SIF in mode I for CT specimens (RL–TL and TR).

In fact, for a longitudinal crack, SENT specimens seem quasi-insensitive to orthotropy: the calibration valid for an isotropic material can be used. In other directions, the difference depends on the species considered.

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RESUME

Comment appliquer les données de la mécanique de la rupture classique pour matériaux isotropes à l'étude de la propagation des fissures dans le bois

L'orthotropie du matériau bois correspond à six directions principales de propagation de fissure RL, TL; RT, TR et LR, LT (le premier indice correspond à la normale au plan de fissure, le deuxième à la direction de propagation). Pour déterminer les facteurs d'intensité de contraintes en pratique, les éprouvettes utilisées sont associées à une calibration obtenue par calcul numérique sur des milieux isotropes. D'une façon générale, ces calibrations ne sont pas valables pour les milieux fortement orientés tels que le bois.

Dans cet article, différents types d'éprouvettes de traction (double poutre en flexion: DCB, traction compacte: CT, simple entaille latérale en traction: SENT) ont été étudiées numériquement (éléments finis singuliers), pour une dizaine d'essences de bois (masses volumiques comprises entre 0,1 et 1,28 g cm^{-3}*) et pour une fissure située suivant un axe matériel. L'éprouvette SENT, dans les directions RL, TL apparaît indépendante de l'orthotropie du matériau. Dans les directions transversales (LR, LT) elle pourrait être utilisée en doublant sa hauteur. Des différences faibles dépendant de l'essence existent aussi entre les directions TR et RT.*

La calibration de l'éprouvette CT en milieu isotrope ne peut pas être utilisée même pour les directions de fendage RL et TL. Pour ces deux dernières, une relation corrective valable pour toutes les essences utilisées avec une bonne approximation est proposée. Dans la direction TR, une calibration différente est également proposée. Par contre, dans la direction RT, il est suggéré de ne pas utiliser le spécimen sans calibration spécifique au matériau essayé.

L'éprouvette DCB, quant à elle, pourrait être employée dans les directions RL, TL ou TR à l'aide d'une calibration issue de calculs analytiques, fonction explicite des constantes élastiques. Cette éprouvette ne présente pas d'intérêt pratique dans les autres directions de propagation.