Wood Sci Technol (2005) 39: 3–17
DOI 10.1007/s00226-004-0255-3

ORIGINAL

Svetlana Vasic · Ian Smith · Eric Landis

Finite element techniques and models for wood fracture mechanics

Received: 7 July 2002 / Published online: 26 November 2004

Abstract Numerical models for wood fracture and failure are commonly based on the finite element method. Most of these models originate from general theoretical considerations for other materials. This limits their usefulness because no amount of complexity in a model can substitute for lack of an appropriate representation of the physical mechanisms involved. As for other materials, wood fracture and failure models always require some degree of experimental calibration, which can introduce ambiguity into numerical predictions because at present there is a high degree of inconsistency in test methods. This paper explores avenues toward achieving models for wood fracture that are both appropriate and robust.

Introduction

The main postulate of finite element analysis (FEA) is that complex domains can be discretized and represented by an assembly of simpler finite sized elements. This enables description of the global problem via a system of differential equations that account for inter-element compatibility and boundary conditions requirements. FEA can be used to model a large array of physical situations and processes including problems in the domains of continuum mechanics, heat and mass transfer and fluid flow. The concepts, fundamentals and application of FEA are described in many texts (e.g., Bathe 1996; Cook 1995; Zenkiewicz and Taylor 1988, 1989) . Other numerical techniques are often

S. Vasic (✉)
Institute of Physics and Material Science, Department of Material Sciences and Process Engineering, University of Natural Resources and Applied Life Sciences, Vienna, Peter Jordan Strasse 82, 1190, Austria
E-mail: svetlana.vasic@boku.ac.at

I. Smith
Faculty of Forestry and Environmental Management, University of New Brunswick, P.O. Box 44555, Fredericton N.B., E3B 6C2, Canada

E. Landis
Department of Civil and Environmental Engineering, University of Maine, Orono, ME 04469-5711, USA

used to represent solid mechanics problems, e.g., the boundary element method (Aliabadi and Rooke 1993; Brabia 1978), but at this stage at least they are not as fully developed. This paper focuses on stress analysis of fracture problems in wood by FEA methods.

The basic premise of modern engineering is that models can be used to extrapolate beyond the range of test data. Therefore, if complex physical processes and phenomena related to fracture in wood are understood for representative situations, numerical models can be built to represent those processes and phenomena beyond the range of those representative situations. FEA and other numerical analysis techniques can therefore never be a total replacement for experimental observations. They are a powerful adjunct that has to be allied with experimental observation and material characterization.

Linear elastic fracture mechanics models

Linear elastic fracture mechanics (LEFM) models are continuum representations and usually implemented by FEA. The concepts are only applicable for estimation of the load level that will propagate an initially sharp crack. Thus, the concepts are unsuitable for predicting development of cracking, especially for materials that develop toughening mechanisms once cracks begin to grow. This can be quite problematic because wood and wood based materials often embody heterogeneity that affects crack extension and promotes toughening.

For anisotropic materials, the basic LEFM theory has to be modified. Wood is commonly regarded as orthotropic with axes of material symmetry corresponding to longitudinal (*L*), radial (*R*) and transverse (*T*) directions. The *L*–*R*–*T* axis system relates to positions within the tree stem from which any particular piece of wood was cut, with the *L* axis coinciding with the pith. To a good approximation, wood is cylindrically orthotropic. However, in fracture analysis the material is presumed rectilinearly orthotropic for tractability of solutions. This amounts to assuming the value of the radial coordinate is large, i.e., the wood is cut well away from the pith. This is often not the case. It is still generally accepted that numerical prediction of crack propagation in orthotropic materials is far more complicated than for the isotropic case due to the relevance of crack orientation, material stiffness properties and material strength properties, which can all have an impact on the direction and load level at which cracks propagate (Boone et al. 1987). No consensus has been reached yet concerning which fracture criterion is best for simulating crack propagation in anisotropic materials. LEFM concepts have been traditionally applied because the techniques are well developed for other materials and not because they are necessarily appropriate.

For homogeneous orthotropic material with a crack lying on one plane of symmetry the stress intensity factors (*K* values) are evaluated according to Sih et al. (1965) and applied within the equation for crack growth $K = K_c$ where K_c is the appropriate fracture toughness. K_c values are considered to be material constants that can be obtained from the experiments with the relationship $K_c = \sqrt{G_c E*}$, where G_c is the critical energy release rate and E^* is the harmonic elastic modulus. Orthotropic stress intensity factors, unlike their isotropic cousins, depend on the elastic constants (Bowie and Freese 1972). Othotropic LEFM relies on an assumption of self-similar crack growth, which is questionable for wood and wood-based materials (Vasic 2000). When a material is not a homogeneous continuum at cellular or finer scales, it should be treated as heterogeneous (Kanninen et al. 1977). There is, therefore, a strong element of educated judgment in any decision to apply LEFM to wood.

Numerical computations in LEFM usually involve crack opening displacements (CODs) using special crack tip elements and/or energy methods. The displacement hybrid finite element developed by Atluri et al. (1975) yields very accurate results but complexity of its formulation has limited its application. Special crack tip elements contain a strain field singularity according to theoretical requirements (Irwin 1957). Foschi and Barrett (1976) used the well-known Sih et al. (1965) displacement field at the tip of a crack in an anisotropic body to enrich an isoparametric quadratic quadrilateral element. The approach was also applied to V-notches in orthotropic plates by Lum and Foschi (1988) . Barsoum (1975) and Henshell and Shaw (1975) independently demonstrated that the inverse square root singularity characteristic of LEFM can be produced using two-dimensional (2D) eight-noded isoparametric (Q8) finite elements with the mid-side nodes adjacent to the crack tip placed at the near quarter points. The same logic applies for a 3D prism. Barsoum (1976) showed that the triangular element formed by collapsing one side of the Q8 element led to far better results than with the rectangular element. The element has been shown to contain an $r^{-1/2}$ singularity, providing a stress field in accordance with the theoretical LEFM stress singularity (Saouma and Schwemmer 1984). This element contains rigid body movements and constant strain modes and thus satisfies the necessary conditions for the convergence. The use of nine-point Gaussian integration for plane elements and 27-point integration for 3D elements provides satisfactory evaluation of the stiffness matrix, thus allowing stress computations for locations very close to the crack tip and hence better evaluation of stress intensity values.

In the case of the triangular isoparametric Barsoum elements the solution for stress-intensity factors has the form:

$$\left\{ \begin{array}{c} K_{\mathrm{I}} \\ K_{\mathrm{II}} \\ K_{\mathrm{III}} \end{array} \right\} = [B]^{-1}[A]\sqrt{\frac{\pi}{2L_1}} \tag{1}$$

where L_1 is the length of the singularity element (Fig. 1), matrix A is a function of node displacements (in the 2D case it reduces to 4 $u_{\mathrm{B}}-u_{\mathrm{c}}$ and 4 $v_{\mathrm{B}}-v_{\mathrm{c}}$), while $[\mathrm{B}]^{-1}$ contains the displacement field shape functions as given for an orthotropic body by Sih et al. (1965). The u and v values are nodal displacements in the plane of face ADF.

A standard finite element program with quadratic isoparametric elements can be modified to extract stress-intensity factors with a rather simple scheme (Boone et al. 1987). This involves:

- Modifying the element stiffness matrix to include orthotropic stiffness constants
- Placing quarter-point elements at the crack tip
- Extracting displacements from the quarter-point elements at the crack faces
- Including a simple algorithm to interpret stress-intensity factors from the displacements

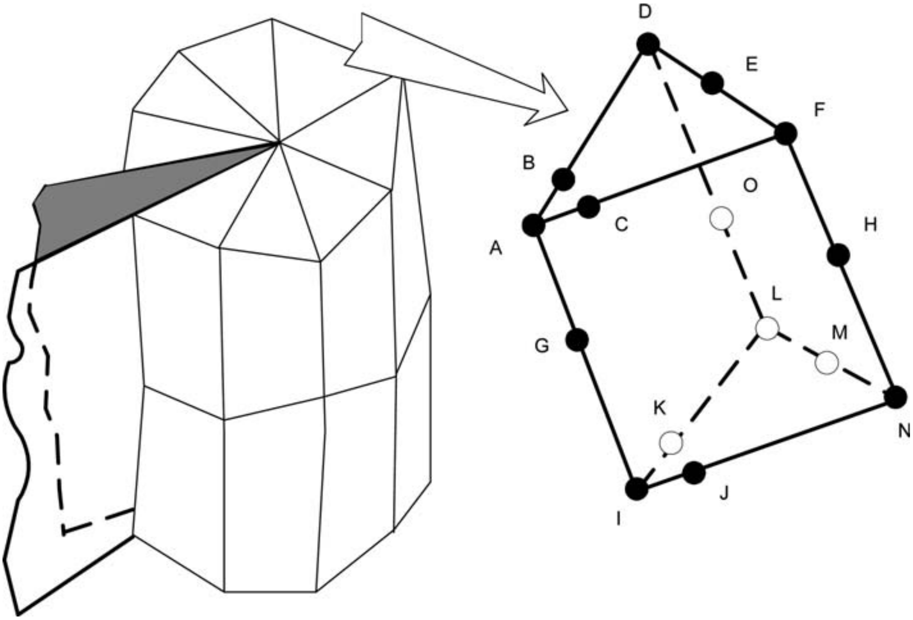

Fig. 1 Barsoum's 3D singular finite element

Accurate computation is attained when the elements are regularly shaped and well distributed around the crack tip. Barsoum elements and this type of procedure have been applied in a number a wood fracture problems including splitting of compact tension (CT) specimens (Stanzl-Tschegg et al. 1995, Valentin and Adjanohoun 1992); splitting of end-tapered double cantilever beam (DCB) and modified CT specimens (Vasic and Smith 2000, 2002); and, failure of beams with notches and holes (Aicher et al. 1995; Smith et al. 1996). In these cases Barsoum's technique was successfully used as the basis of calculations for LEFM parameters (K , K_c and G_c values).

Energy release rates, G values, can be carried out with commercial FEA packages using the J-integral (J) method. Under LEFM conditions $G = J$. Rice (1968) proposed the J-integral as an approximate technique that bypasses the detailed solution of crack boundary value problems. He proved that calculation of J is independent of the integration path used during analysis, with the path enclosing the region (elements) surrounding a crack or notch tip. The average energy density in the J-integral formulation does not depend on the particular stress–strain relation, and therefore the method can be extended to small-scale yielding, including linear work-hardening in elastic–plastic materials. In wood fracture mechanics evaluations, J is a one-step method of estimating G and hence $K = \sqrt{GE*}$ for a given fracture mode (Vasic and Smith 2003). Another computational approach to LEFM is a crack closure integral (Irwin 1958). Irwin's contention was that if a crack extends by a small amount Δc, the energy absorbed in the process is equal to the work required to close the crack to its original length. In terms of an FEA scheme this is one-half the product of the nodal forces and corresponding displacement associated with bringing opposite faces of a crack together. The forces at the crack tip can be obtained by placing very stiff springs at the appropriate nodes and using the forces in the springs as the nodal values. A relatively small amount of computational effort is involved with slight dependence on the assumed crack length. Other possible avenues for computation of LEFM parameters include:

– Weight functions for rectilinearly anisotropic bodies (An 1987)
– Combining weight functions with the virtual crack extension technique (Sha and Yang 1985)
– Energy perturbation for damage tolerant designs (Sha et al. 1988) As yet, none of these techniques have been applied to wood fracture problems.

Whether discrepancies between behaviour of wood and the assumptions that underpin LEFM are important with regard to ability to predict load levels that will fail cracked bodies is problem specific (Smith and Vasic 2003; Smith et al. 2003). As is known from general mechanics considerations, provided geometric proportioning is held constant, the ratio of strain energy stored in a member subjected to external load relative to the energy required for crack extension increases with any increase in the member volume. This means that there is minimal load release when cracks start to propagate and the possibility of crack stabilization is minimal even in the presence of coarse inhomogeneity. Toughening around the crack tip has little influence for large systems and members. Thus, LEFM solutions are asymptotic with true solutions when systems and members are large (Smith and Vasic 2003).

Non-linear elastic fracture models

As already indicated, useful as LEFM can be as an analytical tool, physical processes within the fracture process zone of wood are poorly represented (Smith et al. 2003). Experimental evidence suggests complex fracture and failure mechanisms are operative in wood, especially when phenomena local to a crack are concerned (Smith et al. 2003; Vasic et al. 2002). Non-linear elastic fracture mechanics (NLFM) methods need to be part of an analyst's arsenal. NLFM methods are sophisticated numerical prediction tools that have as their main advantage the ability to predict post-peak stress fracture behaviour.

Fictitious crack model

First attempts to apply NLFM to wood were based on the fictitious crack model (FCM) previously developed to mimic fracture in concrete (Bazant and Planas 1998; Hillerborg et al. 1976; Li and Liang 1994; Rots and de Borst 1988; Yamaguchi and Chen 1990). The FCM is assumed advantageous over LEFM because no pre-existing crack is required and it recognizes modes of energy dissipation other than creation of fracture surface. The concept is that fracturing in a material introduces discontinuities in the displacement field. It is assumed that damage is confined to a fracture plane of zero thickness. FEM implementation links or continuous contact elements are used to connect nodes on opposite faces of existing or potential crack planes (Fig. 2). Linking elements simulate experimental stress vs. crack width relationships (σ–w curves) such as that shown in Fig. 2. There is no need to actually understand the nature of the fracture mechanism as long as suitable σ–w curves can be defined. Hence, the model is fictitious. Many past studies have accepted that the FCM would fit the

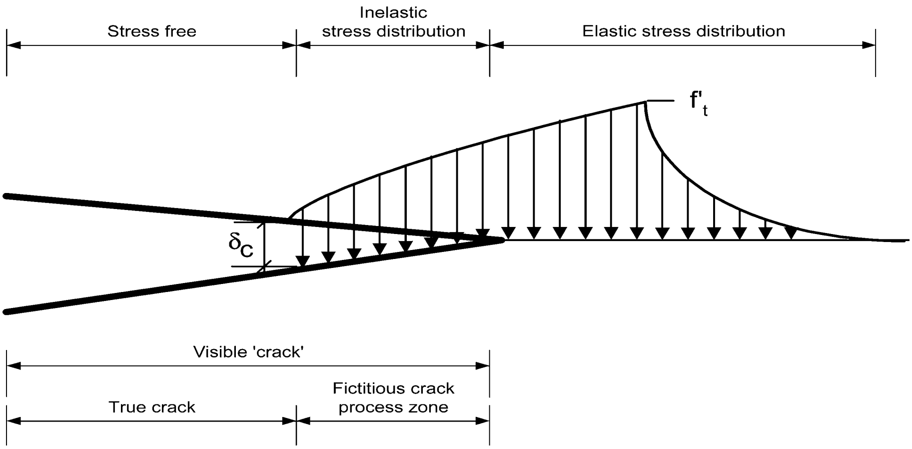

Fig. 2 Fictitious crack model (FCM), f_t is tensile strength, and δ_c is a crack opening

damage processes in wood despite any explicit proof (Vasic 2000). The fracture process zone ahead and behind the crack tip is modelled with springs that have a negative spring constant according to the σ–w curve, since the slope of the bi-linear softening curve is decreasing (Bostrom 1992). Position and direction in which a crack(s) will propagate are assumed in advance and linking elements that are initially very stiff are placed along the anticipated path(s). Stiffness of the linking elements is adjusted ([no tag]softened') according to the specified σ–w curve once the tensile strength has been reached locally. The numerical results are usually presented as a load–displacement curve for a specimen or structural component. It is assumed that once the crack opening is sufficient, spring stiffness drops to zero and no stress transferring ability exists and a real as opposed to fictitious crack is established. The FCM can be applied under combined stress conditions as has been illustrated in the context of adhesive joints that produce softening in wood due to both tension perpendicular to grain and shear parallel to grain analysis (Wernersson 1990).

The thickness effect in CT specimens has been considered using the FCM (Vasic and Smith 2000). The interface finite element method was chosen with a sufficiently fine mesh in the fracture region to minimize discretization errors. Various softening model representations were considered, including bi-linear and multi-linear, and complete load-displacement curves were predicted (Fig. 3). Vasic and Smith (2000) concluded that inconsistencies between experimental and numerical load–crack opening displacements originate from a size-effect in softening curves that are model input, in addition to inadequacy of the model itself in mimicking fracture behaviour of wood. FCM modelling failed to simulate deformations in the crack region to an acceptable accuracy because the assumption that non-linear fracture processes are confined to a plane of zero thickness is incorrect.

Bridged crack model

Based on real-time observation of opening mode fracture processes in soft-woods (Vasic 2000), it has been concluded that a bridged crack model (BCM) is

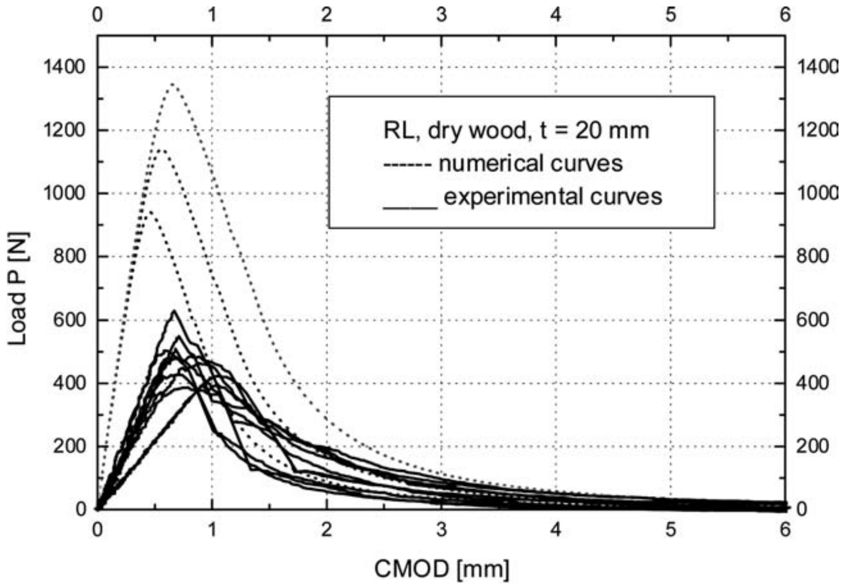

Fig. 3 Application of the FCM to predict load–crack opening displacements

a correct theoretical NLFM representation of wood (Vasic and Smith 2002) . Figure 4 gives a schematic of how the model is implemented. The conceptual difference between FCM and BCM models concerns whether a stress singularity is permitted at the crack tip. The BCM assumes that a stress singularity at a sharp crack tip co-exists with a bridging zone behind the crack tip, i.e., the bridging zone is not fictitious as in the FCM.

The main assumptions of the BCM is that fracture occurs when the critical fracture toughness is reached at the tip of the crack. The criterion for crack

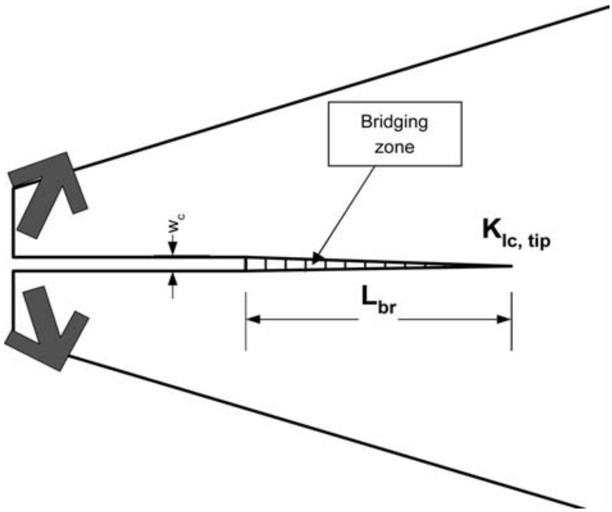

Fig. 4 Bridged crack model (BCM)

extension and opening is therefore the same as for LEFM crack extension. Thus, the fracture criterion is stress based and fracture toughening during crack growth can be represented by simply adding the stress contributed from bridging fibres (or other toughening mechanisms) to the net crack tip stress intensity. The net toughness $K_{net} = K_o + K_b$ where K_o is the intrinsic toughness at the crack tip and K_b the bridging toughness.

Quantitative evaluation of the inelastic contribution K_b is crucial for proper application of the BCM. Results from the quantitative SEM study by Vasic (2000) show that at least for the opening mode, the effect of bridging stresses is to decrease crack opening displacements and shield the crack tip. In a numerical study on tapered DCB fracture specimens by Vasic and Smith (2003), bridging fibres were simulated with the application of parabolic pressure load along the crack faces. The FEM code used had the capability of direct computation of fracture related parameters for orthotropic materials and computation of energy release rates G_{Ic} due to external loading and G_{Ib} due to bridging stresses. Net critical stress intensity factor $K_{Ic\text{-}net}$ was obtained from the energy release rate $G_{Ic\text{-}net}$ using the harmonic orthotropic modulus of elasticity E^*. The concept relates to [no tag]small-scale bridging' where the effect of the stress singularity is strong relative to the effect of bridging stresses. This presumes the size-effect due to bridging to be negligible, as opposed to [no tag]large-scale bridging' (Bao and Suo 1992). The FEM code also had the capability of implicit [no tag]shifting' of nodes in the elements around the crack tip to create a stress singularity. Three-noded CONT2 elements were inserted along the zones expected to be in contact. Elastic orthotropic properties of spruce were assumed based on the values given by Bodig and Jayne (1982). Output from the analysis includes the wedge load P that splits the specimen. Figure 5 shows fracture toughness versus crack length results. As can be deduced, the influence of bridging stresses increases with any increase in the size of the bridging zone if the maximum bridging stress is kept constant. Net and elastic fracture parameters vary with increasing size of the bridging zone, and reach the maximum value when the crack length equals the intrinsic flaw size for the material, which for spruce is 4 mm (Vasic 2000).

Lattice fracture model

This section discusses lattice models as an alternative to the more usual continuum-based representations that are discussed above. Discrete elements within lattice arrangements simulate real ultrastructure features. Therefore, it is straightforward to explicitly incorporate heterogeneity and variability making lattice models a natural choice for representing disordered materials (Curtin and Scher 1990; Herrmann and Roux 1990). It follows that such models can be used to represent wood that embodies both structured and random heterogeneity at various length scales. Being morphology-based the modelling eliminates errors associated with homogenization which occurs in continuum-based FEA. In the past lattice models have been used mainly with concrete-based materials and incorporated both random and uniform lattice geometry with uniform and variable elements (Jirasek and Bazant 1995; Schlangen 1995; Schlangen and Garboczi 1996, 1997).

The material is represented as an array of nodes connected by a network of discrete beam or spring elements. Figure 6 shows one possible discretization

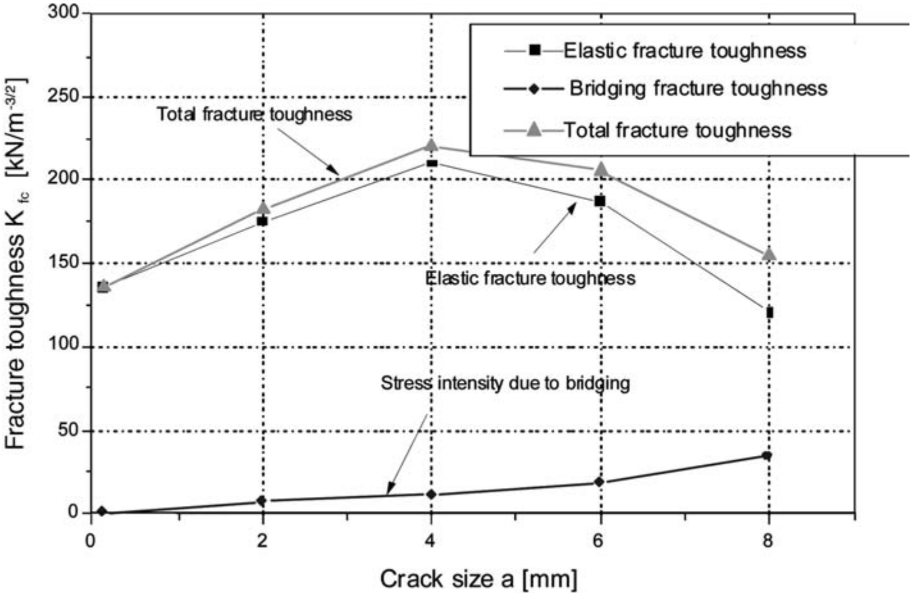

Fig. 5 Fracture toughness vs. crack length for an end-tapered DCB specimen

appropriate for wood. The longitudinal wood cells are represented by beam elements (large horizontal elements in the figure) while a network of diagonal spring elements simulates their connectivity. The chosen size of a lattice cell in the specific example corresponds to a bundle of cells so that the modelling is at the scale of wood growth rings. In general, models may be 2D or 3D and

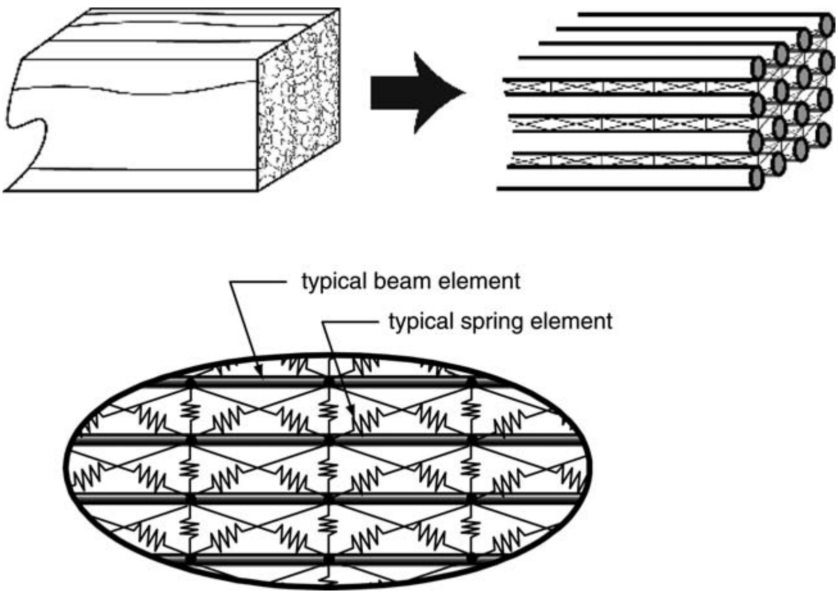

Fig. 6 Lattice finite element mesh for wood

elements defined on any appropriate scale. In order to account for pre-existing heterogeneities, disorder of wood ultrastructure is introduced via statistical variation of element stiffness and strength characteristics. Stiffness and strength characteristics can be assumed to fit a Gaussian (or another) distribution with specified mean and standard deviation. This explicitly introduces the effects of heterogeneity with coefficients of variation typically ranging from 20 to 40% and dependent upon the species (Landis et al. 2002). The model is solved using traditional FEA, calculating element forces and comparing them with previously established strength. Once the strength of any element has been reached, it is considered broken and its contribution to the global stiffness matrix is removed. The system is re-solved under particular displacement or load controlled boundary conditions until criteria for failure of the complete system (specimen) have been satisfied. Clearly, progressive failure is being captured, and damage development is easily followed. Monte Carlo analysis procedures are used to produce a set of pseudo experimental data.

A lattice network for a prismatic tension perpendicular to grain specimen with a rectangular cross-section was developed by the authors. Sensitivity analysis revealed that results stabilize at a beam element length of 0.5 mm and beam element spacing of 0.25 mm. A comparison between numerical and experimental load–displacement curves produced under load control is given in Fig. 7, while a typical simulated damaged lattice is shown in Fig. 8. The lattice shows the main crack being bridged by cells or cell bundles that are responsible for the long tail on the load–displacement curve. This and dispersion of damage throughout a specimen are also characteristic of real specimens. Redundancy in lattice networks allows for the possibility of multiple load paths and hence stable progressive failure, but does not preclude the possibility of unstable failure, which is consistent with experimental observations (Vasic 2000). Similar conclusions follow from simulation of the single-edge notched tension specimen (Landis et al. 2002).

Lattice element properties are not chosen arbitrarily. As elaborated by Davids et al. (2003), element properties are determined from matching the global lattice response to the orthotropic elastic properties of wood in bulk. The

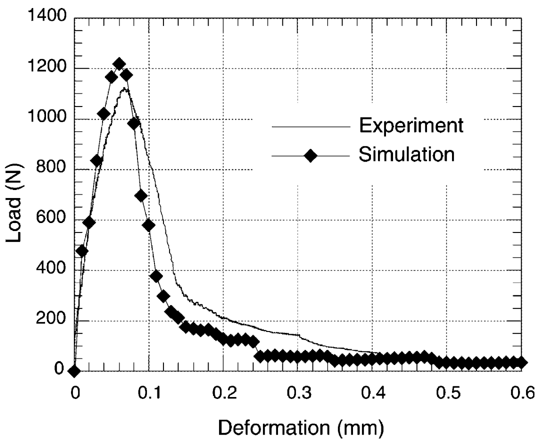

Fig. 7 Lattice fracture model vs. experimental tension perpendicular to grain response under displacement control

Fig. 8 Typical lattice damage pattern tension perpendicular to grain response under displacement control

parameters of the model that can be adjusted are element aspect ratio, the angle that defines the orientation of the diagonal members, and the mean stiffness of each type of element. Optimal mean values of elastic constants are assumed to be those that minimize the normalized least-squares objective function of the orthotropic bulk wood values. Other properties such as mean strengths and coefficients of variation are determined from an adjustment procedure that matches experimental and nominal numerical bulk wood response in shear parallel to grain, radial tension perpendicular to grain and tension parallel to grain.

The research effort in developing this numerical framework of LFM is still in progress and numerous issues are yet to be resolved before the approach can achieve its full potential. Like all other fracture models applied to wood the LFM does not yet recognize that wood embodies both structured and unstructured inhomogeneity.

Non-linear fracture models

There exist several commercial finite element programs specifically aimed at fracture analysis of structural components. A brief discussion of what is available is included here because the programs have capabilities potentially useful in analysis of fracture in wood or wood-based composites. These numerical tool boxes go beyond LEFM and prediction of crack initiation.

The FRANC fracture modelling code was developed by researchers at Cornell University (Wawrzynek and Ingraffea 1985) and can analyse both 2D and 3D problems with hydraulic fracture as the primary application target. FRANC3D provides a mechanism for simulation of arbitrary non-planar cracks along with re-meshing that follow crack growth using LEFM and NLFM. It relies on a hierarchy of topological models for multiple, non-planar, arbitrary shaped cracks which can be on surfaces, interfaces or the interior. The geometry of a specimen evolves over the range of an analysis. The common procedure for incremental numerical simulation of crack propagation is: (1) accurate extraction of stress intensity factors along an arbitrary three-dimensional front; (2) determination of the direction of extension and the crack growth increment; and (3) including the effect of neighbouring features such as interaction with other cracks, boundaries and material interfaces.

The MERLIN fracture modelling code developed by the researchers at the University of Colorado (Saouma et al. 1994) is primarily intended for application to reinforced concrete and dams. This code also has 2D and 3D capabilities, including LEFM and NLFM. The feature for simulation of crack propagation is available with the choice of discrete and smeared crack models. Constitutive models encompass orthotropic materials. Both static and dynamic analyses are feasible. In 3D analysis stress intensity factors and energy release rates are computed based on the domain integral method. A crack propagation feature of special significance to fracture analysis in wood is the ability to handle mixed-mode conditions.

General comments

It is clear from the above that few fracture models were developed with behaviour of wood in mind, and that available models have quite different capabilities. LEFM and derivative modelling concepts are those most commonly adopted. They require an initial crack(s) of predetermined size, location and orientation to be specified and are capable of predicting what level of loading will cause the crack to propagate. If one ignores structured and random inhomogeneity in the material and assumes wood is a continuum it is usually quite easy to select where an initial crack will be located because there will be strong stress concentrations. In two dimensions, the orientation can be assumed along the grain because of the strong elastic anisotropy, but in three dimensions the orientation can be difficult to predict. Suitable initial crack lengths, or crack from shapes in three dimensions, can be judged based on test observations and experience. Experience shows that LEFM and derived models successfully predict the failure load when general dimensions of a component are large compared to the fracture process zone. Although the ratio of the fracture process zone to the initial crack length can also be an issue, LEFM and derived concepts are usually sufficient for predictions associated with global behaviour of [no tag]structural sized members' and complete structural systems (Smith et al. 2003).

Fictitious crack, bridging crack and lattice modelling concepts predict crack evolution as well as propagation loads for components. In the cases of FCM and BCM it is still necessary to predict the location and orientation of any cracks, but not to assign them an initial length. Such models can encompass the transition from an artificial (cut or otherwise machined) crack to a natural crack that mobilizes toughening mechanisms like fibre bridging. Lattice fracture models are the only ones to date capable of predicting unconstrained crack initiation, propagation and evolution. As matters stand, the drawbacks with LFM are that there is no established process for assigning element stiffness and strength properties that is independent of calibration processes, and computational demands can be large. It is anticipated however that both these limitations will be mitigated within the near future. In the longer term LFM should be developed to the stage where it can handle issues such as slow crack growth, load cycling and effects of moisture movements in wood. Experience shows that adoption of more complex models like FCM and BCM is necessary when the stress field developed around a crack tip interacts with high stresses produced by features other than cracks, e.g. gluelines and mechanical fasteners (Smith et al. 2003). As already implied, LFM is only likely to be applied under special circumstances.

Fracture experiments and modelling for wood have focussed on the effects of so-called static loads (monotonic loading that causes failure in about 0.1 h or less), even though such load conditions are rarely, if ever, encountered in practice. Thus, future work needs to focus on extending knowledge and concepts to other loading regimes and time scales. Consideration of longer time scales also means that factors such as the effects of moisture and perhaps temperature should not be ignored.

Conclusion

Analysts have available to them a broad range of fracture modelling concepts that can be applied to wood components and implemented via FEA. Whether it is essential that a chosen model mimics the real failure processes in wood depends on the problem to be analysed and the nature of predictions that are required. The most important factors are that the capabilities of each modelling concept are known and that an appropriate choice is made for the problem at hand. This paper is aimed at achieving these ends.

References

Aicher SM, Schmidt S, Brunold S (1995) Design of timber beams with holes by means of fracture mechanics. Proceedings of the international council for research and innovation in construction—working commission W18 (CIB-W18), meeting 28, Copenhagen, Denmark

Aliabadi MH, Rooke DP (1993) Advances in boundary element methods for fracture mechanics. Series: computational mechanics. Wessex Institute of Technology, Southampton, UK

An D (1987) Weight function theory for rectilinear anisotropic body. Int J Fract 34:85–109

Atluri SN, Kobayashi AS, Nakagaki M (1975) A finite element program for fracture analysis of composite material. In: Fracture mechanics of composites. ASTM STP 593, Philadelphia, PA

Bao G, Suo Z (1992) Remarks on crack-bridging concepts. Appl Mech Rev ASME 45(8):355–366

Barsoum RS (1975) Application of quadratic isoparametric finite elements in linear fracture mechanics. Int J Fract 10:603–605

Barsoum RS (1976) On the use of isoparametric finite elements in linear fracture mechanics. Int J Numer Methods Eng 10:25–37

Bathe KJ (1996) Finite element procedures. Prentice Hall, New Jersey, USA

Bazant ZP, Planas J (1998) Fracture and size effect in concrete and other quasibrittle materials. CRC Press, Boca Raton

Bodig J, Jayne BA (1982) Mechanics of wood and wood composites. Van Nostrand Reinhold, New York

Boone TJ, Wawrzynek PA, Ingraffea AR (1987) Finite element modelling of fracture propagation in orthotropic materials. Eng Fract Mech 26(2):185–201

Bostrom L (1992) Method for determination of the softening behaviour of wood and applicability of a non-linear fracture mechanics model. Lund University, TVBM-1012, Lund, Sweden

Bowie OL, Freese CE (1972) Central crack in plane orthotropic rectangular sheet. Int J Fract Mech 8(1):49–58

Brabia CA (1978) The boundary element for engineers. Pentech Press, London

Cook RD (1995) Concepts and applications of finite element analysis. Wiley & Sons, New York, pp 336

Curtin W, Scher H (1990) Brittle fracture of disordered materials: a spring network model. J Mater Res 5(3):535–553

Davids WG, Landis EN, Vasic S (2003) Lattice models for the prediction of load-induced failure and damage in wood. Wood Fiber Sci 35(1):120–135

Foschi RO, Barrett JD (1976) Stress intensity factors in anisotropic plates using singular isoparametric elements. Int J Numer Mech Eng 10(6):1281–1287

Henshell RD, Shaw KG (1975) Crack tip finite elements are unnecessary. Int J Numer Methods Eng 9(3):495–507

Herrmann HJ, Roux S (1990) Statistical models for the fracture of disordered media. North Holland, Amsterdam

Hillerborg A, Modeer M, Petterson PE (1976) Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. Cem Concr Res 6:773–782

Irwin GR (1957) Analysis of stresses and strains near the end of a crack traversing a plate. J Appl Mech 24:361–364

Irwin GR (1958) Fracture. In Handbuch der Physik, vol VI. Springer, Berlin Heidelberg New York

Jirasek M, Bazant Z (1995) Macroscopic fracture characteristics of random particle systems. Int J Fract 69:201–228

Kanninen MF, Rybicki EF, Brinson HF (1977) A critical look at current applications of fracture mechanics to the failure of fibre-reinforced composites. Composites 8:17–22

Landis EN, Vasic S, Davids WG, Parrod P (2002) Coupled experiments and simulations of microstructural damage in wood. Exper Mech 42(4):389–394

Li YN, Liang RY (1994) Peak load determination in linear fictitious crack model. J Eng Mech ASME 120(2):232–249

Lum C, Foschi RO (1988) Arbitrary V-notches in orthotropic plates. ASCE J Eng Mech 114(4):638–655

Rice RJ (1968) A path independent integral and the approximate analysis of strain concentration by notches and cracks. J Appl Mech 6:379–386

Rots JG, de Borst R (1988) Analysis of mixed-mode fracture of concrete. ASCE J Eng Mech 113(11):1739–1758

Saouma V, Cervenka J, Reich R (1994) MERLIN finite element program. University of Colorado, http://www.ceae.colorado.edu/~saouma/merlin. Cited 11 November 2004

Saouma VE, Schwemmer D (1984) Numerical evaluation of the quarter-point crack tip element. Int J Numer Methods Eng 20:1629–1641

Schlangen E (1995) Experimental and numerical analysis of fracture processes in concrete. Research report, Technical University of Delft, Delft, The Netherlands

Schlangen E, Garboczi E (1996) New method for simulating fracture using an elastically uniform random geometry lattice. Int J Eng Sci 34(10):1131–1144

Schlangen E, Garboczi E (1997) Fracture simulations of concrete using lattice models: computational aspects. Eng Fract Mech 57(2/3):319–332

Sha GT, Yang CT (1985) Weight function calculations for mixed-mode fracture problems with the virtual crack extension technique. Eng Fract Mech 21(6):1119–1149

Sha GT, Chen JK, Yang CT (1988) Energy perturbation finite element technique for damage tolerant design applications. Eng Fract Mech 29(2):197–218

Sih GC, Paris PC, Irwin GR (1965) On cracks in rectilinear anisotropic bodies. Int J Fract 1:189–203

Smith I, Vasic S (2003) Fracture behaviour of softwood. Mech Mater 35:803–815

Smith I, Tan DM, Chui YH (1996) Critical reaction forces for notched timber members. Proceedings of the international wood engineering conference, New Orleans, Louisiana State University, Baton Rouge, USA, pp 3460–3471

Smith I, Landis E, Gong M (2003) Fracture and fatigue in wood. Wiley, Chichester, UK

Stanzl-Tschegg SE, Tan DM, Tschegg EK (1995) New splitting method for wood fracture characterization. Wood Sci Technol 29:31–50

Valentin G, Adjanohoun G (1992) Applicability of classical isotropic fracture mechanics specimens to wood crack propagation studies. Mater Struct 25:3–13

Vasic S (2000) Applications of fracture mechanics to wood. PhD Thesis, University of New Brunswick, Fredericton, NB, Canada

Vasic S, Smith I (2000) Non-linear fracture mechanics analysis of thickness effect in green wood. Proceedings of the world conference of timber engineering 2000, Whistler, Canada, July 31–August 3

Vasic S, Smith I (2002) Bridging crack model for fracture of spruce. Eng Fract Mech 69:745–760

Vasic S, Smith I (2003) Contact-crack problem with friction in spruce. Holz Roh Werkst 61:182–186

Vasic S, Smith I, Landis EN (2002) Fracture zone characterization—micro- mechanical study. Wood Fiber Sci 34(1):42–56

Wawrzynek PA, Ingraffea AR (1985) FRANC software. Cornell University, http://www.cfg.cornell.edu. Cited 11 November 2004

Wernersson H (1990) Fracture characterization of wood adhesive joints. Report TVSM-1006, Lund University, Division of Structural Mechanics, Lund, Sweden

Yamaguchi E, Chen WF (1990) Cracking model for finite element analysis of concrete materials. ASCE J Eng Mech 116(6):1242–1260

Zienkiewicz OC, Taylor RL (1988) The finite element method: vol 1 - basic formulation and linear problems. McGraw Hill, London

Zienkiewicz OC, Taylor RL (1989) The finite element method: vol 2, solid and fluid mechanics, dynamics and non-linearity. McGraw Hill, London