Three-Dimensional Viscoelastic Model with Nonconstant Coefficients

By M. Fafard,[1] M. T. Boudjelal,[2] B. Bissonnette,[3] and A. Cloutier[4]

Abstract: This paper presents a fully 3D viscoelastic model to predict the creep and relaxation behavior of anisotropic materials. This model is based on a phenomenological approach using internal variables and is applicable to nonconstant coefficients. The analytical solution of the set of thermodynamic equations is presented using the reduced time approach in conjunction with modal space. The particular case of isotropic material is presented. In addition, from the general 3D model, the analytical solution in 1D is derived and a connection with the classical rheological model is made. Finally, the model is calibrated and assessed with creep test data for concrete in tension and a parameter sensitivity analysis is performed.

INTRODUCTION

Creep and relaxation models are generally based on a rheological approach. Such an approach, relying on unidimensional considerations, is simple and it offers great flexibility to define different phenomena (Byfors 1980; Bazant et al. 1989a,b, 1997a,b; Coussy and Ulm 1995). Uniaxial creep tests or relaxation tests can be performed to identify all parameters of the rheological model (Bissonnette and Pigeon 1996). Generally, only axial stress-strain information is necessary for this identification. Those models can easily be extended to a multidimensional context for isotropic materials (De Borst and van de Boogaard 1994; Bazant et al. 1997a,b).

Generally, when using a rheological model for an isotropic material such as concrete, only axial strain or axial stress needs to be measured to identify creep or relaxation model parameters. However, one can also monitor the transverse strain to identify the viscoelastic Poisson's ratio (Neville et al. 1983; Li and Qian 1992). This information seems to be superfluous in the case of classical rheological models such as the Kelvin-Voigt chain.

It becomes more difficult to use the same kind of approach in the case of orthotropic materials such as wood. A uniaxial rheological model can be used independently in each principal direction, and using uniaxial test results, parameters of the model can be identified. In the context of multidimensional analysis, it could, however, be inappropriate to use a rheological approach because different physical principles are likely to be violated (Bazant et al. 1997b). Some authors used rheological models in which parameters were identified through appropriate tests allowing the identification of viscoelastic coefficients for the deviatoric and volumetric parts (Pluvinage 1992). But for the orthotropic case, it does not allow for coupled effects between each direction; in fact, multidimensional effects are not taken into account in the model or in the test setup.

Considerable efforts have been devoted over the last 3 decades to understanding and developing realistic 3D constitutive equations for the delayed behavior of concrete [e.g., Bazant (1975, 1988), Huet et al. (1982), Wittmann (1982), Neville et al. (1983), and Bazant et al. (1989a,b, 1997a,b)] and wood [e.g., Morgan et al. (1982), Salin (1992), Luhmann and Niemz (1993), Lin and Cloutier (1996), Mauget and Perré (1996), and Svensson (1996)]. The fundamental origin of creep for concrete still remains not very well understood today. Recent attempts were made to explain the creep phenomenon of concrete in an accurate way. However, models elaborated within a unidimensional framework are limited to some particular aspects of concrete behavior and are lacking in generality (Coussy and Ulm 1995; Granger 1996; Guenot 1996). In addition, the mechanism of creep of concrete, which is of physicochemical nature, occurs at the microscale frame, smaller than the observable scale, and leads to intuitive and nonquantitative explanations (Bazant et al. 1997a,b).

The constitutive modeling of creep and relaxation presented in this paper will be treated within the framework of the thermodynamic of continuous media with internal variables. In addition to the stress or strain tensors, those variables are needed to describe the dissipation energy of the material. The internal variables approach based on the thermodynamic of irreversible processes were first used in continuum mechanics by Biot (1954), Schapery (1964, 1968), and Coleman and Gurtin (1967) and were further developed by Valanis (1966, 1972), Lubliner (1972), and others. Because more variables are introduced, more constitutive relations need to be provided in the form of rate equations representing the evolution of the internal variables. Internal variables may be physical variables (Coussy and Ulm 1995; Guenot 1996), which represent a physicochemical process, or phenomenological variables, which do not have a direct relation to the microphysics and micromechanics of the material behavior but are, in general, easily measurable quantities (Bazant et al. 1989a,b, 1997a,b). It is with the second group that the application of internal variables has been more successful. In this respect, the form of dependence between the strain or stress and the internal variables and the form of the internal variable evolution equations are given in the general framework of the phenomenological approach, which is inspired from the micromechanics of material science.

This paper addresses the methodology used in the development of a 3D viscoelastic model for isotropic, orthotropic, and anisotropic materials. For the formulation of the model, the thermodynamic framework of continuous mechanics based on the internal variables approach is applied. It is assumed that the parameters to be identified can be a function of humidity and temperature (time-dependent parameters). The analytical solution of the thermodynamic system is presented using the concept of reduced time to transform nonconstant coefficients to constant ones in the equivalent time domain (Schapery 1964; Valanis 1972). In the following, emphasis

[1]Prof., Groupe Interdisciplinaire de Recherche en Éléments Finis and Civ. Engrg. Dept., Laval Univ., Quebec, Canada G1K 7P4.

[2]Postdoctoral Fellow, Groupe Interdisciplinaire de Recherche en Éléments Finis and Civ. Engrg. Dept., Laval Univ., Quebec, Canada G1K 7P4.

[3]Res. Assoc., Centre de Recherche Interuniversitaire sur le Béton and Civ. Engrg. Dept., Laval Univ., Quebec, Canada G1K 7P4.

[4]Prof., Groupe Interdisciplinaire de Recherche en Éléments Finis and Wood and Forest Sci. Dept., Laval Univ., Quebec, Canada G1K 7P4.

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will be put on the isotropic case, but all equations are valid for the orthotropic as well as the anisotropic cases.

The 3D model will be simplified to the uniaxial case for relaxation and creep test parameters identification. A calibrating method will be briefly discussed, and some experimental results from concrete tensile creep tests will be treated. A parameter sensitivity analysis will be performed to highlight the effect of some parameters of the model on the results.

THERMODYNAMIC FRAMEWORK AND EVOLUTION LAWS

In this section, the thermodynamic formulation approach to model the delayed behavior of materials is presented. Fundamental equations are well known and are thus introduced without any preliminary consideration. This contribution focuses on the analytical solution of the thermodynamic equation system. Based on concepts of continuum mechanics and irreversible thermodynamics with internal variables, the Clausius-Duhem inequality with respect to the actual state of a viscoelastic continuum is given by

$$\boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}} - \dot{\psi}(\boldsymbol{\varepsilon}, \mathbf{q}_\alpha) \geq 0 \tag{1}$$

where $\boldsymbol{\sigma}$ = second-order stress tensor; $\boldsymbol{\varepsilon}$ = second-order strain tensor; and ψ = Helmholtz free energy. The free energy state of a continuum is completely defined by a set of external variables $\boldsymbol{\varepsilon}$ and a set of internal variables $\mathbf{q}_\alpha$ ($\alpha = 1, \ldots, n$), which will be represented by a second-order tensor. The time derivation of Helmholtz free energy and the substitution into (1) gives

$$\left(\boldsymbol{\sigma} - \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}}\right):\dot{\boldsymbol{\varepsilon}} - \frac{\partial\psi}{\partial\mathbf{q}_\alpha}:\dot{\mathbf{q}}_\alpha \geq \mathbf{0} \tag{2}$$

The inequality (2) must be satisfied for any thermodynamic process. According to the local state law (Lemaitre and Chaboche 1990), the Clausius-Duhem inequality [(2)] leads to the following state equations:

$$\boldsymbol{\sigma} = \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}}; \quad \overline{\mathbf{A}}_\alpha = \frac{\partial\psi}{\partial\mathbf{q}_\alpha} \tag{3}$$

where $\overline{\mathbf{A}}_\alpha$ = thermodynamic force associated with internal variables $\mathbf{q}_\alpha$. In the case of nonreversible behavior of the material, the Clausius-Duhem inequality becomes

$$-\frac{\partial\psi}{\partial\mathbf{q}_\alpha}\frac{d\mathbf{q}_\alpha}{dt} \geq 0 \tag{4}$$

Assuming the linearity of the dissipative mechanism, the so-called dissipative potential Φ can be chosen as a quadratic form of its arguments (Valanis 1972; Lemaitre and Chaboche 1990; Coussy 1995). This potential represents the rate of mechanical energy dissipated in the continuum. In what follows, one uses vector and matrix notations instead of the tensorial one. Hence

$$\Phi = \frac{1}{2}\sum_\alpha \langle\dot{q}_\alpha\rangle[b_\alpha]\{\dot{q}_\alpha\} \tag{5}$$

where $[b_\alpha]$ = matrix containing nonconstant coefficients to be identified. It is a fourth-order tensor with units of force multiplied by time per unit of area (viscosity). The constitutive equations are then formed from the state equations [(3)] and the complementary evolution law that relates, from a phenomenological point of view, the rate of change of internal variables $d\mathbf{q}_\alpha/dt$ to the thermodynamic force $\overline{\mathbf{A}}_\alpha = \partial\psi/\partial\mathbf{q}_\alpha$, which causes the dissipation. The complementary evolution law can then be written

$$\frac{\partial\Phi}{\partial\dot{\mathbf{q}}_\alpha} + \frac{\partial\psi}{\partial\mathbf{q}_\alpha} = 0 \tag{6}$$

where Φ is defined by (5). The use of (5) and (6) ensures that (4) is satisfied. Furthermore, the following quadratic form of the Helmholtz free energy is assumed for isotropic, orthotropic, or anisotropic materials (Valanis 1972)

$$\psi = \frac{1}{2}\langle\varepsilon\rangle[E]\{\varepsilon\} + \sum_\alpha \langle q_\alpha\rangle[B_\alpha]\{\varepsilon\} + \frac{1}{2}\sum_\alpha \langle q_\alpha\rangle[A_\alpha]\{q_\alpha\} \tag{7}$$

where $[E]$ and $[A_\alpha]$ = symmetric matrices. One assumes that there exist many dissipative mechanisms represented through the viscoelastic behavior of a material (summation index α) and $\{\varepsilon\}$ = vector of six components of the deformation tensor (second-order tensor); $\{q_\alpha\}$ = six component internal variables (second-order tensor) that have the same unit as $\{\varepsilon\}$; and $[E]$, $[A_\alpha]$, and $[B_\alpha]$ = matrices containing constitutive coefficients (fourth-order tensor), which are dependent upon humidity and temperature and hence identified through appropriate tests. It will be assumed that they have the same topology as the elastic tensor $[E]$, its units being that of force by unit area.

The free energy is always positive ($\psi \geq 0$). The first part of (7) is the energy due to the instantaneous deformation. The two other terms represent the change in free energy due to the viscoelastic behavior of the material. If this phenomenon is negligible, the free energy then becomes purely elastic.

The stress vector can be obtained from (3) and (7) and takes the following form:

$$\{\sigma\} = \frac{\partial\psi}{\partial\{\varepsilon\}} = [E]\{\varepsilon\} + \sum_\alpha [B_\alpha]^T\{q_\alpha\} \tag{8}$$

where $\{q_\alpha\}$ should be known to estimate stresses. From (5)–(7), a set of equations can be written for each value of α

$$[b_\alpha]\{\dot{q}_\alpha\} + [A_\alpha]\{q_\alpha\} = -[B_\alpha]\{\varepsilon\} \tag{9}$$

It will be assumed in this paper that nonconstant coefficients of matrices $[E]$, $[A_\alpha]$, $[B_\alpha]$, and $[b_\alpha]$ can be defined by

$$[A_\alpha(H, T)] = [\overline{A}_\alpha]f_\alpha(H, T); \quad [B_\alpha(H, T)] = [\overline{B}_\alpha]g_\alpha(H, T) \tag{10a,b}$$

$$[b_\alpha(H, T)] = [\overline{b}_\alpha]h_\alpha(H, T); \quad [E(H, T)] = [\overline{E}]e(H, T) \tag{10c,d}$$

where H and T = humidity and absolute temperature, respectively. One assumes that there exist reference matrices with constant coefficients corresponding to a reference state (matrices with overbars). At this state, scalar functions are equal to 1. Those functions are difficult to identify, and it will be assumed that $f_\alpha = g_\alpha$ as a first approach.

GENERALIZED VISCOELASTIC CONSTITUTIVE EQUATIONS

Considering (10) together with (9), one may write

$$\frac{h_\alpha}{f_\alpha}[\overline{b}_\alpha]\{\dot{q}_\alpha\} + [\overline{A}_\alpha]\{q_\alpha\} = -[\overline{B}_\alpha]\{\varepsilon\} \tag{11}$$

The stresses corresponding to the steady-state solution are

$$\{\sigma\} = \left(e(H, T)[\overline{E}] - \sum_\alpha f_\alpha(H, T)[\overline{B}_\alpha]^T[\overline{A}_\alpha]^{-1}[\overline{B}_\alpha]\right)\{\varepsilon\} = [E_\infty]\{\varepsilon\} \tag{12}$$

where $[E_\infty]$ = matrix containing elastic parameters for a long-term loading. Matrices $[\overline{E}]$ and $[\overline{A}_\alpha]$ are symmetric and positive definite and thus $\|[E_\infty]\| \leq \|[\overline{E}_\alpha]\|$, where $\|\cdot\|$ is a matrix norm. Furthermore, it can be observed in the case of constant strain loading that stresses will start from instantaneous deformation and decrease to reach a long-term stress level smaller than the initial one. Thus, between these two limits, the model should be able to predict the variation of stresses for constant strains or, conversely, the variation of strains for constant stresses.

To find a solution to the viscoelastic transient problem, (11) must be solved. If all coefficients of this equation system are constants, it is easy to solve it. Otherwise, the concept of equivalent time will be used to find the analytical solution. The internal variables $\{q^a\}$ at $t = 0$ are equal to 0; thus, matrix $[E]$ represents the instantaneous elastic tensor upon loading. A standard elastic test can be used to identify the coefficients of this matrix at the initial reference state. In the same way, a stationary state can be used to identify matrix $[E_\alpha]$.

It can also be observed that equations presented in this subsection are valid for any anisotropic material. For the isotropic case, two coefficients per matrix are unknown, whereas for orthotropic materials such as wood, nine coefficients are unknown. It will be shown in the following that, for the isotropic case, the solutions of (11) will require the identification of four coefficients for each value of α.

Analytical Solution

Eq. (11) is a system of six differential equations that can easily be solved analytically if all coefficients are constant. If coefficients are not constant, a new variable $\chi(t)$ representing a reduced or equivalent time can be introduced, and rewriting (11)

$$\left\{\frac{dq_\alpha}{dt}\right\} = \left\{\frac{dq_\alpha}{d\chi_\alpha}\right\}\frac{d\chi_\alpha}{dt} = \{q'_\alpha\}\frac{d\chi_\alpha}{dt}, \quad \text{with } \frac{d\chi_\alpha}{dt} = \frac{f_\alpha}{h_\alpha} \tag{13}$$

$$[\bar{b}_\alpha]\{q'_\alpha\} + [\bar{A}_\alpha]\{q_\alpha\} = -[\bar{B}_\alpha]\{\varepsilon\} \tag{14}$$

In what follows, the prime notation means a derivative with respect to the new variable $\chi(t)$. The latter is an equivalent time, which can be defined

$$\chi_\alpha(t) = \int_0^t \Theta_\alpha(\tau)\, d\tau; \quad \Theta_\alpha(t) = \frac{f_\alpha(t)}{h_\alpha(t)} \tag{15}$$

The function $\Theta_\alpha(t)$ should be identified by conducting appropriate laboratory tests. It is now easy to solve (14) using a diagonalization technique (modal projection)

$$\{q_\alpha\} = \{x_\alpha\}e^{-\lambda_\alpha\chi_\alpha}; \quad \{q'_\alpha\} = -\lambda_\alpha\{x_\alpha\}e^{-\lambda_\alpha\chi_\alpha} \tag{16}$$

where $\{x_\alpha\}$ and λ_α = eigenvectors and eigenvalues, respectively, of the following system of equations:

$$([\bar{A}_\alpha] - \lambda_\alpha[\bar{b}_\alpha])\{x_\alpha\} = 0; \quad [\bar{A}_\alpha][X_\alpha] = [\bar{b}_\alpha][X_\alpha][D_\alpha] \tag{17}$$

where $[D_\alpha]$ = diagonal matrix containing eigenvalues λ_α (six for the anisotropic case); and $[X_\alpha]$ = matrix where each column contains eigenvectors corresponding to the eigenvalues. Those eigenvalues are real and positive because matrices $[\bar{A}_\alpha]$ and $[\bar{b}_\alpha]$ are considered symmetric and positive definite. For the isotropic case, there exist two distinct eigenvalues, whereas for the orthotropic case, there are six eigenvalues for each α value. The unit of λ_α is that of a frequency (1/time).

Given the properties of eigenvectors, one can write

$$[X_\alpha]^{-1}[\bar{A}_\alpha][X_\alpha] = [D_2] = [D_1][D_\alpha] \tag{18a}$$

$$[X_\alpha]^{-1}[\bar{b}_\alpha][X_\alpha] = [D_1] \tag{18b}$$

where $[D_1]$ and $[D_2]$ = diagonal matrices. Defining new variables in modal space

$$\{q_\alpha\} = [X_\alpha]\{z_\alpha\} \tag{19}$$

Eq. (14) is rewritten

$$\{z'_\alpha\} + [D_\alpha]\{z_\alpha\} = \{F_\alpha\} \tag{20a}$$

where

$$\{F_\alpha\} = -\left[\frac{1}{D_1}\right][X_\alpha]^{-1}[\bar{B}_\alpha]\{\varepsilon\} \tag{20b}$$

Because $[D_\alpha]$ = diagonal matrix, (20) represents a set of six uncoupled linear differential equations of the first-order function of equivalent time χ. The solution for each equation (i = 1 to 6) is

$$z_\alpha(i) = \int_0^{\chi_\alpha} e^{-\lambda_\alpha(i)(\chi_\alpha - \chi_{\alpha_\tau})}\mathbf{F}_\alpha(i)\, d\chi_{\alpha_\tau} \tag{21}$$

where $\mathbf{F}_\alpha(i)$ = ith component of the vector $\mathbf{F}_\alpha$. Using (15), (19), and (20), the solution for $\{q_\alpha\}$ in real space can be written

$$\begin{Bmatrix} q_\alpha(1) \\ q_\alpha(2) \\ \vdots \\ q_\alpha(6) \end{Bmatrix} = -[X_\alpha]\begin{Bmatrix} \dfrac{\langle x_\alpha(1)\rangle[\bar{B}_\alpha]}{D_1(1)}\left(\int_0^t \Theta_\alpha e^{-\lambda_\alpha(1)(\chi_\alpha(t)-\chi_\alpha(\tau))}\{\varepsilon\}\, d\tau\right) \\ \dfrac{\langle x_\alpha(2)\rangle[\bar{B}_\alpha]}{D_1(2)}\left(\int_0^t \Theta_\alpha e^{-\lambda_\alpha(2)(\chi_\alpha(t)-\chi_\alpha(\tau))}\{\varepsilon\}\, d\tau\right) \\ \vdots \\ \dfrac{\langle x_\alpha(6)\rangle[\bar{B}_\alpha]}{D_1(6)}\left(\int_0^t \Theta_\alpha e^{-\lambda_\alpha(6)(\chi_\alpha(t)-\chi_\alpha(\tau))}\{\varepsilon\}\, d\tau\right) \end{Bmatrix} \tag{22}$$

where $\langle x_\alpha(i)\rangle$ = ith row of matrix $[X]^{-1}$; and $\lambda_\alpha(i)$ = ith eigenvalue of the system [(17)]. Stresses have been defined by (8). With internal variables defined in (22), stresses can be written

$$\{\sigma\} = [E]\{\varepsilon\} - \sum_\alpha f_\alpha \int_0^t [H^d_\alpha]\{\varepsilon\}\, d\tau \tag{23a}$$

$$[H^d_\alpha(t - \tau)] = \Theta_\alpha[\bar{B}_\alpha]^T[X_\alpha][D_1]^{-1}[D_{\text{exp}}][X_\alpha]^{-1}[\bar{B}_\alpha] \tag{23b}$$

$$[D_{\text{exp}}] = \begin{bmatrix} e^{-\lambda_\alpha(1)(\chi_\alpha(t)-\chi_\alpha(\tau))} & & & \\ & e^{-\lambda_\alpha(2)(\chi_\alpha(t)-\chi_\alpha(\tau))} & & \\ & & \ddots & \\ & & & e^{-\lambda_\alpha(6)(\chi_\alpha(t)-\chi_\alpha(\tau))} \end{bmatrix} \tag{23c}$$

3D ISOTROPIC CASE

In the previous section, the general solution of the viscoelastic problem for the 3D anisotropic case was presented. The equations can be simplified in the case of an isotropic material such as concrete. For this purpose, because the dissipative part is assumed isotropic and according to the elastic case, the following assumption is introduced: the topology of all matrices is similar to that of the elastic matrix. This means that, for each matrix, there are two unknown coefficients analogous to the elastic modulus and Poisson's coefficient. Using the following generic isotropic matrix:

$$[P(E_p, \beta_P)] = \frac{E_p}{(1 - 2\beta_P)(1 + \beta_P)} \cdot \begin{bmatrix} 1-\beta_P & \beta_P & \beta_P & 0 & 0 & 0 \\ & 1-\beta_P & \beta_P & 0 & 0 & 0 \\ & & 1-\beta_P & 0 & 0 & 0 \\ & \text{symm.} & & \dfrac{(1-2\beta_P)}{2} & 0 & 0 \\ & & & & \dfrac{(1-2\beta_P)}{2} & 0 \\ & & & & & \dfrac{(1-2\beta_P)}{2} \end{bmatrix} \tag{24}$$

the matrices $[\bar{A}_\alpha]$, $[\bar{B}_\alpha]$, $[\bar{b}_\alpha]$, and $[\bar{E}]$ can be defined by

$$[\bar{A}_\alpha] = [P(E_{A_\alpha}, \mu_\alpha)]; \quad [\bar{B}_\alpha] = [P(E_{B_\alpha}, \zeta_\alpha)] \qquad (25a,b)$$

$$[\bar{b}_\alpha] = [P(E_{b_\alpha}, \xi_\alpha)]; \quad [\bar{E}] = [P(E_E, \nu)] \qquad (25c,d)$$

Using these assumptions, it can be shown (with symbolic software such as MAPLE) that there exist only two distinct eigenvalues for the equations system [(17)]

$$\lambda_{1_\alpha} = \frac{E_{A_\alpha}}{E_{b_\alpha}} \frac{1 - 2\xi_\alpha}{1 - 2\mu_\alpha}; \quad \lambda_{2_\alpha} = \frac{E_{A_\alpha}}{E_{b_\alpha}} \frac{1 + \xi_\alpha}{1 + \mu_\alpha} \qquad (26)$$

With these eigenvalues, it can be shown that (23b) becomes

$$[H^d_\alpha] = \begin{bmatrix} H^d_{\alpha 11} & H^d_{\alpha 12} & H^d_{\alpha 12} & 0 & 0 & 0 \\ & H^d_{\alpha 11} & H^d_{\alpha 12} & 0 & 0 & 0 \\ & \text{symm.} & H^d_{\alpha 11} & 0 & 0 & 0 \\ & & & H^d_{\alpha 44} & 0 & 0 \\ & & & & H^d_{\alpha 44} & 0 \\ & & & & & H^d_{\alpha 44} \end{bmatrix} \qquad (27a)$$

$$H^d_{\alpha 11} = \Theta_\alpha \left[K^d_\alpha \lambda_{1_\alpha} e^{-\lambda_{1_\alpha}(\chi_\alpha(t) - \chi_\alpha(\tau))} + \frac{4}{3} G^d_\alpha \lambda_{2_\alpha} e^{-\lambda_{2_\alpha}(\chi_\alpha(t) - \chi_\alpha(\tau))} \right] \qquad (27b)$$

$$H^d_{\alpha 12} = \Theta_\alpha \left[K^d_\alpha \lambda_{1_\alpha} e^{-\lambda_{1_\alpha}(\chi_\alpha(t) - \chi_\alpha(\tau))} - \frac{2}{3} G^d_\alpha \lambda_{2_\alpha} e^{-\lambda_{2_\alpha}(\chi_\alpha(t) - \chi_\alpha(\tau))} \right] \qquad (27c)$$

$$H^d_{\alpha 44} = \Theta_\alpha \lambda_{2_\alpha} e^{-\lambda_{2_\alpha}(\chi_\alpha(t) - \chi_\alpha(\tau))} \qquad (27d)$$

$$2G^d_\alpha = H^s_\alpha \Phi_\alpha = \frac{E^d_\alpha}{(1 + \nu^d_\alpha)} \qquad (27e)$$

$$3K^d_\alpha = H^s_\alpha \Psi_\alpha = \frac{E^d_\alpha}{(1 - 2\nu^d_\alpha)} \qquad (27f)$$

$$H^s_\alpha = \frac{(E_{B_\alpha})^2}{E_{A_\alpha}}; \quad \Psi_\alpha = \frac{(1 - 2\mu_\alpha)}{(1 - 2\zeta_\alpha)^2}; \quad \Phi_\alpha = \frac{1 + \mu_\alpha)}{(1 + \zeta_\alpha)^2} \qquad (27g\text{–}i)$$

where the dissipative coefficients E^d_α, ν^d_α, G^d_α, and K^d_α are introduced. They correspond respectively to the dissipative Young's modulus, Poisson's ratio, dissipative shear modulus, and dissipative compressibility modulus. Equations defined in (27) show that the two eigenvalues (or the relaxation time if one takes the inverse of the eigenvalues) are related to K^d_α, λ_{1_α}, λ_{2_α}, and G^d_α, which means that dissipation energy due to creep or relaxation is associated with both hydrostatic and shear behavior. These four parameters, along with instantaneous shear and compressibility coefficients, could be identified by conducting either tensile or compressive creep tests combined with torsion creep tests. Such experiments performed under different conditions of exposure would allow the identification of functions $\Theta_\alpha(T, H)$, $f_\alpha(T, H)$, and $e(T, H)$.

Lemaitre and Chaboche (1990) have proposed a viscoelastic model with two relaxation times for the isotropic case. They consider that K^d and G^d are proportional to their elastic counterpart (K and G) through λ_1 and λ_2. Thus, only four coefficients would be identified (λ_1, λ_2, K, and G).

3D ISOTROPIC CASE WITH $\xi_\alpha = \mu_\alpha$

Eq. (27) can be simplified, assuming the existence of only one dissipative mechanism. It means that the eigenvalues defined in (26) are equal and thus $\xi_\alpha = \mu_\alpha$

$$\lambda_{\alpha_1} = \lambda_{\alpha_2} = \frac{E_{A_\alpha}}{E_{b_\alpha}} = \lambda_\alpha \qquad (28)$$

$$H^d_{\alpha 11} = \gamma_\alpha \left[K^d_\alpha + \frac{4}{3} G^d_\alpha \right] = \gamma_\alpha \frac{E^d_\alpha(1 - \nu^d_\alpha)}{(1 - 2\nu^d_\alpha)(1 + \nu^d_\alpha)} \qquad (29a)$$

$$H^d_{\alpha 12} = \gamma_\alpha \left[K^d_\alpha - \frac{2}{3} G^d_\alpha \right] = \gamma_\alpha \frac{E^d_\alpha \nu^d_\alpha}{(1 - 2\nu^d_\alpha)(1 + \nu^d_\alpha)} \qquad (29b)$$

$$H^d_{\alpha 44} = \gamma_\alpha G^d_\alpha; \quad \gamma_\alpha = \Theta_\alpha \lambda_\alpha e^{-\lambda_\alpha(\chi_\alpha(t) - \chi_\alpha(\tau))} \qquad (29c,d)$$

This is consistent with the classical Kelvin-Voigt rheological model.

1D VISCOELASTIC MODEL

In this section, viscoelastic equations for 1D creep tests in tension or compression are presented. It is assumed that test specimens are cylindrical and that axial as well as lateral strains are measured. Equations for both relaxation and creep tests are dealt with. In the following, all coefficients are assumed to be independent of time, temperature, and humidity, which means that $\Theta_\alpha = 1$ and thus that $\chi = t$. In addition, the existence of only one dissipative mechanism is assumed ($\alpha = 1$).

Relaxation Case

In the case of a uniaxial relaxation test, the axial strain is imposed and only the corresponding stress is nonzero. From (23a) and (27b–i) one can write

$$\sigma_1 = E_{11}\varepsilon_1 + 2E_{12}\varepsilon_r - \int_0^t \left(K^d\lambda_1 e^{-\lambda_1(t-\tau)} + \frac{4}{3} G^d\lambda_2 e^{-\lambda_2(t-\tau)} \right) \varepsilon_1 \, d\tau - 2\int_0^t \left(K^d\lambda_1 e^{-\lambda_1(t-\tau)} - \frac{2}{3} G^d\lambda_2 e^{-\lambda_2(t-\tau)} \right) \varepsilon_r \, d\tau \qquad (30)$$

$$0 = E_{11}\varepsilon_1 + E_{12}(\varepsilon_1 + \varepsilon_r) - \int_0^t \left(K^d\lambda_1 e^{-\lambda_1(t-\tau)} - \frac{2}{3} G^d\lambda_2 e^{-\lambda_2(t-\tau)} \right) (\varepsilon_1 + \varepsilon_r) \, d\tau - \int_0^t \left(K^d\lambda_1 e^{-\lambda_1(t-\tau)} + \frac{4}{3} G^d\lambda_2 e^{-\lambda_2(t-\tau)} \right) \varepsilon_r \, d\tau \qquad (31)$$

where ε_1, σ_1, and ε_r refer respectively to axial strain, axial stress, and lateral strain. The parameters E_{11} and E_{12} are components of matrix $[\bar{E}]$ defined in (25d). Using Laplace transform, these two linear equations can be written in a matrix form

$$\begin{Bmatrix} L(\sigma_1) \\ 0 \end{Bmatrix} = \begin{bmatrix} E_{11} - K^d \frac{\lambda_1}{s + \lambda_1} + \frac{4}{3} G^d \frac{\lambda_2}{s + \lambda_2} & 2E_{12} - 2K^d \frac{\lambda_1}{s + \lambda_1} + \frac{4}{3} G^d \frac{\lambda_2}{s + \lambda_2} \\ E_{12} - K^d \frac{\lambda_1}{s + \lambda_1} + \frac{2}{3} G^d \frac{\lambda_2}{s + \lambda_2} & E_{11} + E_{12} - 2K^d \frac{\lambda_1}{s + \lambda_1} - \frac{2}{3} G^d \frac{\lambda_2}{s + \lambda_2} \end{bmatrix} \begin{Bmatrix} L(\varepsilon_1) \\ L(\varepsilon_r) \end{Bmatrix} \qquad (32)$$

where $L(\cdot)$ represents the Laplace transform operator. Assuming that $\varepsilon_1 = \varepsilon$ is constant over time, it can be shown that from (32)

$$L(\sigma_1) = \left(E_{11} - K^d \frac{\lambda_1}{s + \lambda_1} + \frac{4}{3} G^d \frac{\lambda_2}{s + \lambda_2} \right) \frac{\varepsilon}{s} - \left(2E_{12} - 2K^d \frac{\lambda_1}{s + \lambda_1} + \frac{4}{3} G^d \frac{\lambda_2}{s + \lambda_2} \right) \cdot \frac{\left[E_{12} - K^d \frac{\lambda_1}{s + \lambda_1} + \frac{2}{3} G^d \frac{\lambda_2}{s + \lambda_2} \right]}{E_{11} + E_{12} - 2K^d \frac{\lambda_1}{s + \lambda_1} - \frac{2}{3} G^d \frac{\lambda_2}{s + \lambda_2}} \frac{\varepsilon}{s} \qquad (33)$$

Thus, the solution in the time domain is the inverse Laplace transform of (33).

Creep Case

Starting from (32), one can easily obtain the following relation for the creep model:

$$\begin{Bmatrix} L(\varepsilon_1) \\ L(\varepsilon_r) \end{Bmatrix} = \frac{1}{\det} \begin{bmatrix} E_{11} + E_{12} - 2K^d \dfrac{\lambda_1}{s + \lambda_1} - \dfrac{2}{3} G^d \dfrac{\lambda_2}{s + \lambda_2} & -\left(2E_{12} - 2K^d \dfrac{\lambda_1}{s + \lambda_1} + \dfrac{4}{3} G^d \dfrac{\lambda_2}{s + \lambda_2}\right) \\ -\left(E_{12} - K^d \dfrac{\lambda_1}{s + \lambda_1} + \dfrac{2}{3} G^d \dfrac{\lambda_2}{s + \lambda_2}\right) & E_{11} - K^d \dfrac{\lambda_1}{s + \lambda_1} + \dfrac{4}{3} G^d \dfrac{\lambda_2}{s + \lambda_2} \end{bmatrix} \begin{Bmatrix} L(\sigma_1) \\ 0 \end{Bmatrix} \quad (34)$$

where det = determinant of the matrix defined in (32). Thus, for each deformation component, one has

$$L(\varepsilon_1) = \frac{1}{\det}\left[E_{11} + E_{12} - 2K^d \frac{\lambda_1}{s + \lambda_1} - \frac{2}{3} G^d \frac{\lambda_2}{s + \lambda_2}\right] L(\sigma_1) \quad (35a)$$

$$L(\varepsilon_r) = \frac{-1}{\det}\left[E_{12} - K^d \frac{\lambda_1}{s + \lambda_1} + \frac{2}{3} G^d \frac{\lambda_2}{s + \lambda_2}\right] L(\sigma_1) \quad (35b)$$

Assuming that $\sigma_1 = \sigma$ is independent of time, one can demonstrate that the following equations hold for creep:

$$\varepsilon_1 = \frac{1}{\gamma\beta}\left[\frac{1}{E} - \frac{1}{3}\frac{1}{G}\left(\frac{K^d}{K} + \gamma \frac{G^d}{G} e^{-\lambda_b t}\right) - \frac{1}{9}\frac{1}{K}\left(\frac{G^d}{G} + \beta \frac{K^d}{K} e^{-\lambda_a t}\right)\right]\sigma \quad (36a)$$

$$\varepsilon_r = \frac{1}{\gamma\beta}\left[\frac{-\nu}{E} + \frac{1}{6}\frac{1}{G}\left(\frac{K^d}{K} + \gamma \frac{G^d}{G} e^{-\lambda_b t}\right) - \frac{1}{9}\frac{1}{K}\left(\frac{G^d}{G} + \beta \frac{K^d}{K} e^{-\lambda_a t}\right)\right]\sigma \quad (36b)$$

where

$$\lambda_a = \lambda_1^{\gamma}; \quad \lambda_b = \lambda_2\beta \quad (36c,d)$$

$$\gamma = 1 - \frac{K^d}{K}; \quad \beta = 1 - \frac{G^d}{G} \quad (36e,f)$$

$$G = \frac{E_E}{2(1 + \nu)}; \quad K = \frac{E_E}{3(1 - 2\nu)} \quad (36g,h)$$

Coefficients G and K = instantaneous shear and compressibility modulus, respectively. In addition to elastic coefficients, one has to identify four parameters: G^d, K^d, λ_1, and λ_2. If one makes the hypothesis that $\xi_\alpha = \mu_\alpha$, there will be three parameters to identify in addition to the elastic ones: G^d, K^d, and λ. To obtain the steady-state solution to this problem, coefficients λ_a and λ_b must be positive. With values of λ_1 and λ_2 being positive, γ and β should be positive; thus

$$K^d \le K; \quad G^d \le G \quad (37)$$

At $t = 0$, the elastic behavior is found

$$\varepsilon_1 = \frac{\sigma}{E_E}; \quad \varepsilon_r = -\nu \frac{\sigma}{E_E}$$

thus

$$\nu = -\frac{\varepsilon_r}{\varepsilon_1}$$

When $t \to \infty$, the steady-state solution is

$$\varepsilon_1 = \frac{1}{\gamma\beta}\left[\frac{1}{E} - \frac{1}{3}\frac{1}{G}\frac{K^d}{K} - \frac{1}{9}\frac{1}{K}\frac{G^d}{G}\right]\sigma = \frac{1}{E}\frac{1}{\gamma\beta}\left[1 - \frac{2(1 + \nu)}{3}\frac{K^d}{K} - \frac{(1 - 2\nu)}{3}\frac{G^d}{G}\right]\sigma \quad (38a)$$

$$\varepsilon_r = \frac{-1}{\gamma\beta}\left[\frac{\nu}{E} - \frac{1}{6}\frac{1}{G}\frac{K^d}{K} + \frac{1}{9}\frac{1}{K}\frac{G^d}{G}\right]\sigma = \frac{-1}{E}\frac{1}{\gamma\beta}\left[\nu - \frac{(1 + \nu)}{3}\frac{K^d}{K} + \frac{(1 - 2\nu)}{3}\frac{G^d}{G}\right]\sigma \quad (38b)$$

It can be observed that the ratio $-(\varepsilon_r/\varepsilon_1)$ gives an apparent long-term Poisson's ratio. Eq. (38) demonstrates that this apparent Poisson's ratio can vary with time.

Relationship with Rheological Model

Starting from (36) and assuming that the solid is incompressible ($K = K^d = \infty$), one can show that the creep equations can be rewritten

$$\varepsilon_1 = \frac{1}{E - E^d}\left[1 - \frac{E^d}{E} e^{-\lambda_b t}\right]\sigma \quad (39a)$$

$$\varepsilon_r = -\frac{1}{2}\frac{1}{E - E^d}\left[1 - \frac{E^d}{E} e^{-\lambda_b t}\right]\sigma = -\frac{1}{2}\varepsilon_1 \quad (39b)$$

At time $t = 0$ and $t = \infty$, (39a) gives

$$\varepsilon_1(t = 0) = \frac{\sigma}{E}; \quad \varepsilon_1(t = \infty) = \frac{\sigma}{E - E^d} = \frac{\sigma}{E_\infty} \quad (40a,b)$$

Under the assumptions of material incompressibility and uniaxial creep, the previous constitutive equation may be represented by a three-parameter Kelvin-Voigt rheological model (Fig. 1) with the following relationships:

$$\varepsilon = \frac{1}{E - E^d}\left(1 - \frac{E^d}{E} e^{-\lambda t}\right)\sigma \quad (41a)$$

$$E = E_1; \quad E^d = \frac{E_1^2}{E_1 + E_2}; \quad \lambda = \frac{E_2}{\eta} \quad (41b\text{–}d)$$

where E_1 and E_2 = spring constants; and η = viscosity of the dashpot. At $t = 0$, only elastic deformation takes place and is equal to σ/E_1. The viscosity of the dashpot is fully active at that time, the spring No. 2 (which corresponds to spring constant E_2) is not active yet. At $t = \infty$, the dashpot has dissipated its energy and the stiffness E_2 of the spring is fully active, which leads to a total extraneous deformation equal to σ/E_2.

PARAMETERS CALIBRATION WITH TEST DATA

One of the objectives of this paper was to establish a procedure that enables the identification of the parameters de-

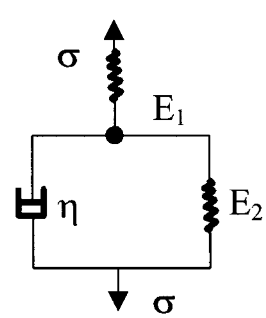

FIG. 1. Three-Parameter Kelvin-Voigt Rheological Model

scribing the constitutive behavior of a viscoelastic material. The identification problem is formulated as an optimization of a functional that is expressed as the difference between experimental and theoretical results using the least-squares method. The functional has to be minimized with respect to the constitutive parameters. Although the calibration presented hereafter used uniaxial creep test results, the constitutive model developed in this paper is applicable in a multiaxial context as well.

TENSILE CREEP OF CONCRETE

Tensile creep can play a beneficial role in reducing the restrained shrinkage stresses induced during the drying process of concrete, especially in repair. The tensile creep can, from a strain balance point of view, counteract the drying shrinkage strain (Bissonnette and Pigeon 1996; Boudjelal et al. 1998). A series of tensile creep tests have been carried out on concrete using a special apparatus developed for this aim (Bissonnette 1996). Only basic creep is considered in this paper.

The 1D viscoelastic model described in (38) is used to model the creep tests. Instead of G_d and K_d, E_d and ν_d will be identified [(27e–i)].

The tensile creep tests of interest for the actual study were performed on sealed specimens at 23 ± 2°C and 50 ± 5% relative humidity after 7 days of curing. The test data used for the actual identification correspond to a concrete mixture having a water-to-cement ratio of 0.40, and the instantaneous Young's modulus at the time of loading was 32,000 MPa. The applied stress was 1.583 MPa for the test starting at 7 days of curing (Bissonnette 1996). As previously mentioned, a least-squares method was used to identify the creep parameters of the proposed uniaxial creep model. The instantaneous Poisson's ratio is taken to be equal to 0.15.

First, a dissipative Poisson's ratio ν_d equal to 0.15 (as in the instantaneous one) was considered. Two cases were considered here to calibrate the parameters of the model: (1) the existence of only one dissipative mechanism is assumed ($\lambda = \lambda_1 = \lambda_2$) (two-parameter model: λ and E_d); and (2) two dissipative mechanisms are assumed to exist ($\lambda_1 \neq \lambda_2$) (three-parameter model: λ_1, λ_2, and E_d). Figs. 2 and 3 show the prediction of creep using the proposed uniaxial model together with the ex-

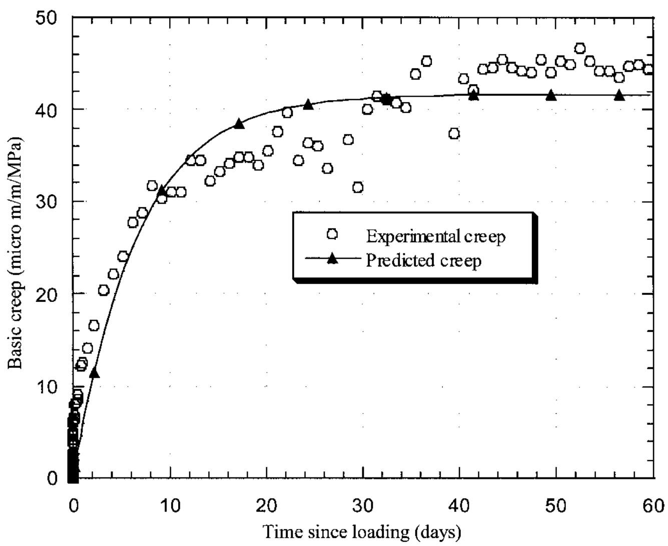

FIG. 2. Basic Concrete Creep in Case of Two Parameters: λ = 0.351 day^{-1} and E^d = 18,288.6 MPa

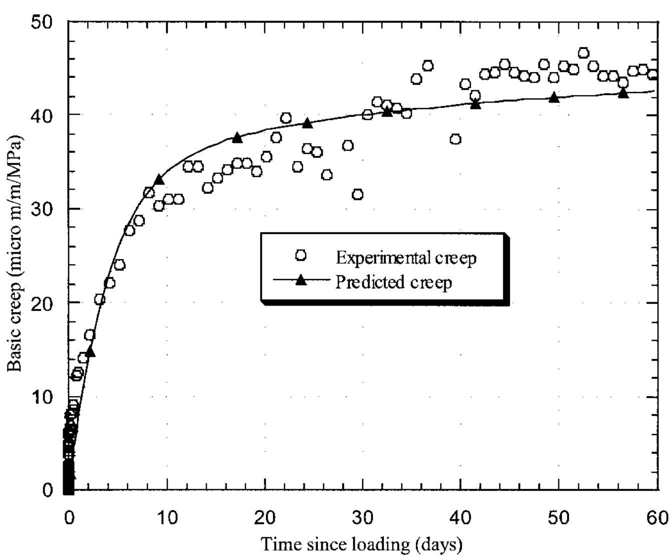

FIG. 3. Basic Concrete Creep in Case of Three Parameters: λ_1 = 0.613 day^{-1}, λ_2 = 0.0675 day^{-1}, and E^d = 18,816.3 MPa

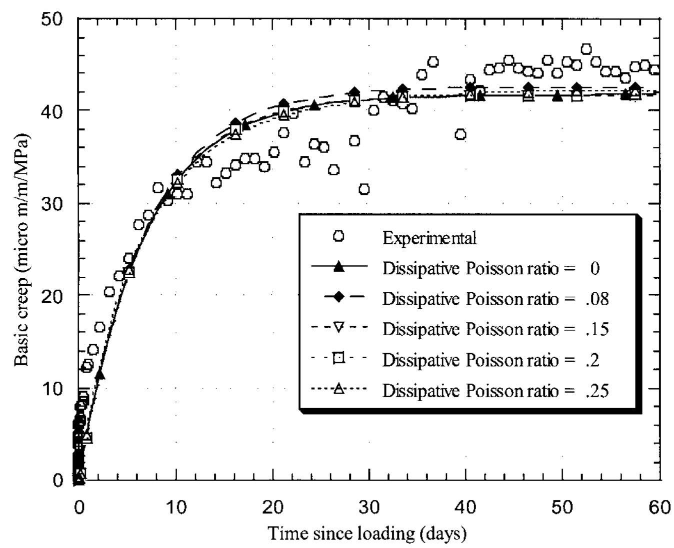

FIG. 4. Basic Creep in Tension in Case of One Dissipative Mechanism for Different Dissipative Poisson's Ratios

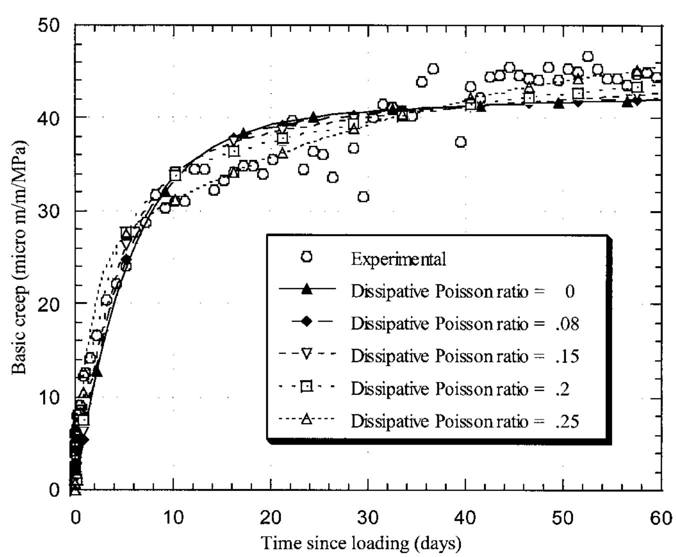

FIG. 5. Basic Creep in Tension in Case of Two Distinct Dissipative Mechanisms for Different Dissipative Poisson's Ratios

TABLE 1. Values of Calibrated Creep Parameters

	$\lambda = \lambda_1 = \lambda_2$		$\lambda_1 \neq \lambda_2$		
ν^d	E^d (MPa)	λ (1/day)	E^d (MPa)	λ_1 (1/day)	λ_2 (1/day)
0.0	16,969.2	0.3664	17,147.0	0.4751	0.0427
0.05	17,910.9	0.3602	17,843.8	0.4975	0.0477
0.08	18,066.5	0.3565	18,215.1	0.5178	0.0519
0.12	18,207.5	0.3524	18,619.9	0.5601	0.0595
0.15	18,288.6	0.3510	18,816.3	0.6129	0.0675
0.18	17,967.3	0.3532	18,864.2	0.7010	0.0784
0.20	16,726.6	0.3582	18,774.9	0.7951	0.0875
0.25	15,908.6	0.3930	17,708.6	1.3603	0.1277

perimental values. It can be assessed from the two figures that the three-parameter model gives better results than the two-parameter model for both the early age and the later stages.

In the list of parameters to be identified, the dissipative Poisson's ratio was not included because the corresponding functional for minimization is very sensitive to it, especially when this value is >0.25. This sensitivity affects the convergence of the calibration technique. To show the effect of this parameter, it was varied from 0 to 0.25 (Figs. 4 and 5 and Table 1). From Fig. 4, it appears that, when the deviatoric part has a dissipative mechanism identical to that of the spherical part, the creep curves are not really affected by the dissipative Poisson's ratio. It means that it is not possible to improve the calibration by varying this parameter. In the case of two distinct dissipative mechanisms, the creep curves seem to be significantly influenced with respect to the dissipative Poisson's ratio; thus, a better fitting can be performed.

CONCLUSIONS

The main contribution of this paper was the development of a 3D viscoelastic constitutive model for anisotropic materials. Using the concept of reduced time, the thermodynamic equation was solved analytically. The solution shows that there exist, in the general case, six relaxation times for each summation index α. Using a classical rheological model, only one relaxation time can be identified.

The general anisotropic equation has been simplified to the isotropic case where there exist two relaxation times associated with the deviatoric and the spherical part of the stress tensor. Using appropriate tests and simplification of the 3D equation of viscoelasticity, an identification of the unknown parameters was performed using creep tests in tension. It has been observed that the model with two relaxation times gives a better approximation of the creep behavior than the one with one relaxation time. The sensitivity analysis presented in this paper indicates the important role of the dissipative Poisson's ratio on the creep model when the deviatoric and spheric dissipative mechanisms are different.

This 3D approach is compatible with the thermodynamic formalism; thus, one can define a general framework to identify creep/relaxation parameters for an orthotropic material such as wood. In future work, the effect of humidity will be taken into account using the equivalent time function. There will also be an attempt to take transversal strain results into consideration to identify parameters including the dissipative Poisson's ratio in the case of isotropic material. Finally, this work will be extended to the modeling of the creep/relaxation of wood in the three principal directions, taking into account the information (strain) from the orthogonal directions.

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NOTATION

The following symbols are used in this paper:

$\overline{\mathbf{A}}_\alpha$ = thermodynamic forces associated with internal variables $\mathbf{q}_\alpha$;
$[b_\alpha]$ = matrix containing nonconstant coefficients associated with dissipative potential;
E = instantaneous Young's modulus;
$[E]$, $[A_\alpha]$, $[B_\alpha]$ = matrices defining free energy constitutive coefficients of free energy;
E_α^d = dissipative Young's modulus;
E_{A_α}, μ_α = constitutive parameters associated with matrix $[A_\alpha]$;
E_{B_α}, ζ_α = constitutive parameters associated with matrix $[B_\alpha]$;
E_{b_α}, ξ_α = constitutive parameters associated with matrix $[b_\alpha]$;
$[E_\infty]$ = matrix containing long-term elastic parameters;
G = instantaneous shear modulus;
G_α^d = dissipative shear modulus;
K = instantaneous compressibility modulus;
K_α^d = dissipative compressibility modulus;
$\mathbf{q}_\alpha$ = set of internal variables;
$\{q_\alpha\}$ = six components of internal variables;
s = Laplace variable;
$\{x_\alpha\}$ = eigenvectors;
$\{z_\alpha\}$ = modal coordinates;
$\boldsymbol{\varepsilon}$ = strain tensor;
$\{\varepsilon\}$ = six components of deformation tensor;
ε_1, ε_r = axial and radial strains;
λ_α (i) = eigenvalues connected to the relaxation time;
ν = instantaneous Poisson's coefficient;
ν_α^d = dissipative Poisson's coefficient;
$\boldsymbol{\sigma}$ = stress tensor;
σ_1, σ_r = axial and radial stresses;
Φ = dissipative potential;
χ_α = equivalent time variable function of real time t; and
Ψ = Helmholtz free energy.