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Physical Relations for Wood at Variable Humidity

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Abstract. The paper discusses the physical relations between wood at a plane state of stress and variable humidity. A simple theory for wetted wood under stress is constructed based on the balance equations and the thermodynamics of irreversible processes. The theoretical curves are compared with experimental curves for pine sapwood wetted at uniaxial and biaxial states of stress. The so-called mechano-sorptive effect, which arises during the simultaneous action of mechanical loading and wetting process, is taken into account. A high consistency of the theoretical and experimental curves is stated.

Key words: swelling of stressed wood, mechano-sorptive effect, physical relations; variable material constants, pine sapwood, theoretical and experimental studies.

1. Introduction

One observes a significant difference in swelling strains of unstressed and stressed water-soaked wood (see, e.g., Kowal *et al.*, 1992; Kowal and Kowalski, 1995). For wood under mechanical loading which is simultaneously wetted, one can differentiate four strain components: instantaneous, swelling, creep, and mechano-sorptive strain.

Instantaneous strain (elastic or elastic-plastic) is produced immediately after the application of the mechanical load. The plastic strains are neglected in our considerations.

The swelling strains arise during the wetting process and are the largest of all those mentioned above.

Creep is the time-dependent portion of the total deformation due to the imposed stress history (see Ranta-Maunus, 1975; Bazant, 1985). In our experiments, the time for one test was about half an hour, which is too short for a significant increase in a creep strain. Therefore, the creep is ignored in our studies.

The mechano-sorptive strain is due to an interaction between stress and moisture content changes. In tension and compression, this effect is seen as a decrease or increase in the amount of swelling associated with moisture content change (see Rybarczyk, 1973; Ganowicz and Rybarczyk, 1974). The mechano-sorptive strain seems to be proportional to the product of stress and moisture content. Such a strain arises in our physical relations when the material constants are considered as functions of moisture content.

This work aims at construction of a simple theory for wetted wood in a stressed state. The theoretical considerations are based entirely on the concept of balance equations of mass, momentum, energy and entropy, and the thermodynamics of irreversible processes. We take into account elastic, swelling and mechano-sorptive strains. The stress–strain–moisture content relation is expressed by the Neo–Hookean constitutive equation whose material moduli vary with moisture content. It means that the mechano-sorptive effect is taken into account.

A characteristic feature of wood is anisotropy of its physical and mechanical properties resulting from wood anatomic structure. We assume wood to be orthotropic and the number of material coefficients which have to be estimated equals 9. This kind of anisotropy is characterized by three mutually perpendicular planes of symmetry, which are assigned according to anatomic directions of wood (see Figure 2(a)).

The experiments were carried out on samples of pine sapwood cut crosswise to the wood fibres. Thus, we were able to estimate the material constants in the radial and tangential to the growth rings directions. The moisture content of the sample was changed from the dry state to the value exceeding the fiber saturation point (approx. 30%). The wetting process was carried out under three different states of stress (see Kowal *et al.*, 1992; Kowal and Kowalski, 1995):

– tension in radial direction,
– compression in tangential direction,
– both of the states of stress mentioned above acting together (biaxial stress).

The final results of this paper are the physical relations describing the mechanical behaviour of stressed wood during the wetting process. The theoretical curves are compared with the experimental curves and there is a good compliance between them.

2. Fundamental Equations

Consider the fluid-saturated wood as consisting of two overlapping continua: the wood fibres (skeleton) and the fluid. The mass of wood fibres (skeleton) is characterized by the bulk density ρ_s [kg/m^3] and the mass of fluid by the bulk density ρ_f [kg/m^3]. We shall also use the notion of specific moisture content defined as $\Theta = \rho_f/\rho_s$. The measurable kinematic quantities in the present theory are the velocity of wood (skeleton) deformation $\mathbf{v}_s$ and the mass discharge of fluid through the surface $\mathbf{w}$[kg/m^2 s]. It will be helpful in our considerations to use an average velocity of fluid flow defined as

$$\mathbf{v}_f \overset{\text{df}}{=} \mathbf{v}_s + \mathbf{w}/\rho_f. \tag{2.1}$$

All these quantities are function of position vector $\mathbf{x}$ and time t.

Let us conceptually separate from the medium an arbitrary three-dimensional control volume $V(t)$ bounded by a regular surface $A(t)$ oriented spatially by an outward directed unit normal vector $\mathbf{n}$. The masses of skeleton and fluid which in

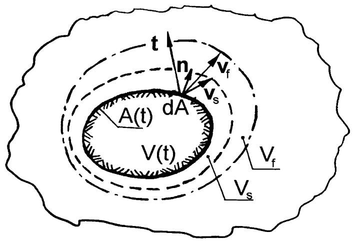

Figure 1. Non-stationary control volume $V(t)$.

time t occupy the volume V will be distributed in time $t + \Delta t$ in volumes V_s and V_f, respectively, which in general differ from each other. Thus, the control volume is non-stationary and changes with velocity $\mathbf{v}_s$ for the skeleton and velocity $\mathbf{v}_f$ for the fluid (Figure 1).

The equations of balance of mass, momentum and energy for the individual constituents can be written as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_s \,\mathrm{d}V = 0, \tag{2.2a}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho_f \,\mathrm{d}V = 0, \tag{2.2b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_s \mathbf{v}_s \,\mathrm{d}V = \int_A \mathbf{t}_s \,\mathrm{d}A + \int_V \rho_s \mathbf{g} \,\mathrm{d}V + \int_V \mathbf{f}_{sf} \,\mathrm{d}V, \tag{2.3a}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho_f \mathbf{v}_f \,\mathrm{d}V = \int_A \mathbf{t}_f \,\mathrm{d}A + \int_V \rho_f \mathbf{g} \,\mathrm{d}V + \int_V \mathbf{f}_{fs} \,\mathrm{d}V, \tag{2.3b}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_s \left(u_s + \frac{1}{2}\mathbf{v}_s \cdot \mathbf{v}_s\right) \mathrm{d}V \;&=\; \int_A (\mathbf{t}_s \cdot \mathbf{v}_s - \mathbf{q}_s \cdot \mathbf{n}) \,\mathrm{d}A \;+\\ &+\; \int_V [\rho_s(\mathbf{g} \cdot \mathbf{v}_s + \mathbf{r}_s) + Q_{sf}] \,\mathrm{d}V,\end{aligned} \tag{2.4a}$$

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho_f \left(u_f + \frac{1}{2}\mathbf{v}_f \cdot \mathbf{v}_f\right) \mathrm{d}V \;&=\; \int_A (\mathbf{t}_f \cdot \mathbf{v}_f - \mathbf{q}_f \cdot \mathbf{n}) \,\mathrm{d}A \;+\\ &+\; \int_V [\rho_f(\mathbf{g} \cdot \mathbf{v}_f + \mathbf{r}_f) + Q_{sf}] \,\mathrm{d}V.\end{aligned} \tag{2.4b}$$

In the above equations $\mathrm{d}/\mathrm{d}t$ and $\mathrm{D}/\mathrm{D}t$ denote the material derivatives with convection velocities $\mathbf{v}_s$ and $\mathbf{v}_f$, respectively, $\mathbf{t}_s$ and $\mathbf{t}_f$ are the stress vectors in the skeleton and fluid referred to the total surface of wet wood, $\mathbf{g}$ is the gravity force, $\mathbf{f}_{sf} = -\mathbf{f}_{fs}$ denotes the interaction forces between the constituents (due to, for example, viscous drag), u_s and u_f denote the internal energy per unit mass of the skeleton and fluid, $\mathbf{q}_s$ and $\mathbf{q}_f$ denote the heat fluxes, r_s and r_f denote the heat supply per unit mass of the skeleton and fluid, and $Q_{sf} = -Q_{fs}$ denote exchange of energy between the constituents.

We write the balance of entropy for the medium as a whole assuming equal temperatures for both constituents:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_s s_s \,\mathrm{d}V + \frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho_f s_f \,\mathrm{d}V = \int_V S_{int}\,\mathrm{d}V - \int_A \left[\frac{\mathbf{q}}{\vartheta}\right]\cdot \mathbf{n}\,\mathrm{d}A + \int_V \frac{\rho r}{\vartheta}\,\mathrm{d}V, \tag{2.5}$$

where s_s and s_f denote the entropies per unit mass of the skeleton and fluid, S_{int} is the internal entropy produced during irreversible processes, $\mathbf{q} = \mathbf{q}_s + \mathbf{q}_f$ is the total heat flux, $\rho r = \rho_s r_s + \rho_f r_f$ is the total heat supply per unit volume, and ϑ is the absolute temperature.

Using Cauchy's formulae for stresses $\mathbf{t}_s = \mathbf{T}_s\mathbf{n}$ and $\mathbf{t}_f = \mathrm{P}_f\mathbf{n}$ (the stress deviator in fluid is neglected) and the Gauss–Ostrogradzky theorem, we can change the surface integrals into volume integrals and after some reductions write the balance equations in a local form:

$$\frac{\mathrm{d}\rho_s}{\mathrm{d}t} + \rho_s \,\mathrm{div}\,\mathbf{v}_s = 0, \tag{2.6a}$$

$$\frac{\mathrm{D}\rho_s}{\mathrm{D}t} + \rho_f \,\mathrm{div}\,\mathbf{v}_f = 0, \tag{2.6b}$$

$$\rho_s\frac{\mathrm{d}\mathbf{v}_s}{\mathrm{d}t} = \mathrm{div}\,\mathbf{T}_s + \rho_s\mathbf{g} + \mathbf{f}_{fs}, \tag{2.7a}$$

$$\rho_f\frac{\mathrm{D}\mathbf{v}_f}{\mathrm{D}t} = \mathrm{grad}\,\mathrm{P}_f + \rho_f\mathbf{g} + \mathbf{f}_{fs}, \tag{2.7b}$$

$$\rho_s\frac{\mathrm{d}u_s}{\mathrm{d}t} = \mathbf{T}_s\,\mathrm{grad}\,\mathbf{v}_s - \mathbf{f}_{sf}\cdot\mathbf{v}_s - \mathrm{div}\,\mathbf{q}_s + \rho_s r_s + Q_{sf}, \tag{2.8a}$$

$$\rho_f\frac{\mathrm{D}u}{\mathrm{D}t} = \mathrm{P}_f\,\mathrm{div}\,\mathbf{v}_f - \mathbf{f}_{fs}\cdot\mathbf{v}_f - \mathrm{div}\,\mathbf{q}_f + \rho_f r_f + Q_{fs}, \tag{2.8b}$$

$$\rho\frac{\mathrm{d}s_s}{\mathrm{d}t} + \rho_f\frac{\mathrm{D}s_f}{\mathrm{D}t} = S_{int} - \mathrm{div}\frac{\mathbf{q}}{\vartheta} + \frac{\rho r}{\vartheta}. \tag{2.9}$$

Considering the wetting processes of wood, it is convenient to formulate the field equations in terms of material coordinates of the solid matrix. Therefore, we reformulate the balance equations in such a way that all the thermodynamic functions are referred to the mass of the solid (skeleton) body (see Kowalski, 1996). Such an approach provides a rational way for construction of a more practical model of wetting because the boundary is clearly defined and formulation of boundary conditions causes no difficulties (see Kowalski and Strumiłło, 1997).

We rearrange the balance equations using operators and functions referred to the solid frame, omitting the accelerations of the skeleton and fluid, and writing the equations of momentum and energy for the medium as a whole. The final form of such reformulated balance equations is

$$\dot{\rho}_s + \rho_s \operatorname{div} \mathbf{v}_s = 0, \tag{2.10a}$$

$$\rho_s \dot{\Theta} = -\operatorname{div} \mathbf{w}, \tag{2.10b}$$

$$\operatorname{div} \mathbf{T} + \rho \mathbf{g} \cong 0, \qquad \mathbf{T} = \mathbf{T}_s + P_f \mathbf{I}, \qquad \rho = \rho_s + \rho_f, \tag{2.11}$$

$$\rho_s \dot{u} = \mathbf{T}\mathbf{E} + \rho_s \dot{\Theta} h_f - \mathbf{w} \cdot \operatorname{grad} h_f + \mathbf{w} \cdot \mathbf{g} - \operatorname{div} \mathbf{q} + \rho r, \tag{2.12}$$

$$-\rho_s \left(\dot{f} + s\dot{\vartheta} - \mu\dot{\Theta} - \frac{1}{\rho_s}\mathbf{T}\dot{\mathbf{E}} \right) - \mathbf{w} \cdot \operatorname{grad} \hat{\mu} - \frac{\mathbf{q} + \mathbf{w} s_f \vartheta}{\vartheta} \cdot \operatorname{grad} \vartheta \geqslant 0, \tag{2.13}$$

where a dot over the symbol denotes the time derivative $\mathrm{d}/\mathrm{d}t$, $u = u_s + \Theta u_f$, $s = s_s + \Theta s_f$ and $f = u - s\vartheta$ denote the internal energy, the entropy and the free energy of wet wood referred to unit mass of the skeleton (dry basis), $h_f = u_f - P_f/\rho_f$ is the moisture enthalpy, $\mu = h_f - s_s\vartheta$ is the moisture potential (chemical plus capillary potential), $\hat{\mu} = \mu + \mu_g$ is the total moisture potential (chemical, capillary and gravitational potentials together) with $\mathbf{g} = -\operatorname{grad}\mu_g$, $\dot{\mathbf{E}} \cong [\operatorname{grad} \mathbf{v}_s + (\operatorname{grad} \mathbf{v}_s)^T]$ is the strain rate tensor of wood.

The second law of thermodynamics, stating that the entropy produced in the system is always positive:

$$S_{int} \geqslant 0, \tag{2.14}$$

was used and thus the inequality (2.13) is obtained. For each reversible process all fluxes disappear. In such a case the entropy produced in the system equals zero and the inequality (2.13) becomes equality.

We assume that the free energy is a function of parameters of state such as the strain tensor $\mathbf{E}$, the moisture content Θ and the temperature ϑ, that is

$$f = f(\mathbf{E}, \Theta, \vartheta). \tag{2.15}$$

Substituting the time derivative of this function into (2.13) just for the case of the reversible process one finds

$$\mathbf{T} = \rho_s \frac{\partial f}{\partial \mathbf{E}} = \mathbf{T}(\mathbf{E}, \Theta, \vartheta), \tag{2.16}$$

$$s = -\frac{\partial f}{\partial \vartheta} = s(\mathbf{E}, \Theta, \vartheta), \tag{2.17}$$

$$\mu = \frac{\partial f}{\partial \Theta} = \mu(\mathbf{E}, \Theta, \vartheta). \tag{2.18}$$

These are the equations of state. Assuming the equations of state to be right for both reversible and irreversible processes, we get the residual inequality of the form

$$-\mathbf{w} \cdot \text{grad}\, \hat{\mu} - \frac{\mathbf{q} + \mathbf{w} s_f \vartheta}{\vartheta} \cdot \text{grad}\, \vartheta \geqslant 0. \tag{2.19}$$

One can find a family of admissible solutions which satisfy the inequality (2.19). We choose the simplest solution, which is the sufficient condition satisfying this inequality, namely (see Kowalski *et al.*, 1997):

$$\begin{aligned} \mathbf{w} = {} & -\mathbf{L}_{\text{m}} \,\text{grad}\, \hat{\mu} = -(-\mathbf{C}_{\text{E}} \,\text{grad}\, E + \mathbf{C}_{\text{m}} \,\text{grad}\, \Theta + \\ & + \mathbf{C}_{\text{T}} \,\text{grad}\, \vartheta - \mathbf{L}_{\text{m}} \mathbf{g}), \quad \mathbf{L}_{\text{m}} \geqslant 0, \end{aligned} \tag{2.20}$$

$$\mathbf{q} + \mathbf{w} s_f\, \vartheta = -\mathbf{L}_{\text{T}} \,\text{grad}\, \vartheta, \quad \mathbf{L}_{\text{T}} \geqslant 0. \tag{2.21}$$

Thus, the flux of mass is proportional to the gradient of moisture potential or to the gradients of volume deformation $E = \text{tr}\mathbf{E}$, moisture content Θ, and temperature ϑ. The chain rule expansion for the gradient of moisture potential was used in (2.20). The introduced tensorial coefficients, $\mathbf{L}_{\text{m}}$, $\mathbf{C}_{\text{E}}$, $\mathbf{C}_{\text{m}}$, $\mathbf{C}_{\text{T}}$ have to be estimated in experimental studies. Recalling the individual thermodynamic forces and measuring the discharge of fluid through the surface, we can calculate the suitable coefficients. This was not, however, the aim of our present experimental studies.

The heat flux is proportional to the gradient of temperature. Note, that in isothermal case (ϑ = const) the heat flux equals to the heat transported by the mass flux, i.e. $\mathbf{q} = -\mathbf{w} s_f\, \vartheta$.

3. Physical Relations

We assume that the relation between the mechanical strains and the stresses takes the form of Neo-Hookean relation with material constants dependent on the moisture content and on the temperature of the material. Developing the free energy function (2.15) in Taylor series in the neighbourhood of free swelling strain $\overset{\circ}{\mathbf{E}}(\Theta, \vartheta)$, retaining terms not higher than second order and availing of the state equation (2.16), we obtain

$$\mathbf{T} = \rho_{\text{s}} \frac{\partial f(\overset{\circ}{\mathbf{E}}, \Theta, \vartheta)}{\partial \mathbf{E}} + \overset{(4)}{\mathbf{C}}(\mathbf{E} - \overset{\circ}{\mathbf{E}}), \tag{3.1}$$

where

$$\overset{(4)}{\mathbf{C}} = \rho_{\text{s}} \frac{\partial f(\overset{\circ}{\mathbf{E}}, \Theta, \vartheta)}{\partial \mathbf{E} \otimes \partial \mathbf{E}}, \tag{3.2}$$

is the tensor of stiffness describing the material properties of wood. It depends on the moisture content and the temperature. We assume that the free swelled state is free of stresses. Then, the first term in (3.1) equals zero. In subscripts notation the relation (3.1) takes the form

$$\mathbf{T}_{ij} = \mathbf{C}_{ijkl}(E_{kl} - \overset{\circ}{E}_{kl}). \tag{3.3}$$

One can reverse this relation with respect to strain and write:

$$E_{ij}(T_{kl}, \Theta, \vartheta) = \overset{\circ}{E}_{ij}(\Theta, \vartheta) + A_{ijkl}(\Theta, \vartheta)T_{kl}, \tag{3.4}$$

where $\overset{\circ}{E}_{ij}(\Theta, \vartheta)$ is the free swelling strain and $A_{ijkl}(\Theta, \vartheta)$ is the tensor of susceptibility dependent on the moisture content and the temperature.

We have carried out our experiments in isothermal conditions. Therefore, the tensor of susceptibility is taken to be dependent on the moisture content in the following way:

$$A_{ijkl}(\Theta, \vartheta) = A_{ijkl}(\Theta_\circ, \vartheta) + B_{ijkl}(\Theta_\circ, \vartheta)(\Theta - \Theta_\circ), \tag{3.5}$$

where $A_{ijkl}(\Theta_\circ, \vartheta)$ is the tensor of elastic susceptibility responsible for the instantaneous strains at the moisture content $\Theta = \Theta_\circ$ and tensor $B_{ijkl}(\Theta_\circ, \vartheta)$ will be termed as that responsible for the mechano-sorptive strains. Thus, the structure of our physical relations is

$$\begin{aligned} &E_{ij}(T_{kl}, \Theta, \vartheta) \\ &\quad = \overset{\circ}{E}_{ij}(\Theta, \vartheta) + A_{ijkl}(\Theta_\circ, \vartheta)T_{kl} + B_{ijkl}(\Theta_\circ, \vartheta)T_{kl}(\Theta - \Theta_\circ). \end{aligned} \tag{3.6}$$

In this paper the coefficients will be estimated for the plane state of stress.

It is assumed that the structure of wood is symmetrical with respect to the three planes assigned by the anatomic directions of wood, namely: lengthwise L, radial R, and tangential T, as it is shown in Figure 2.

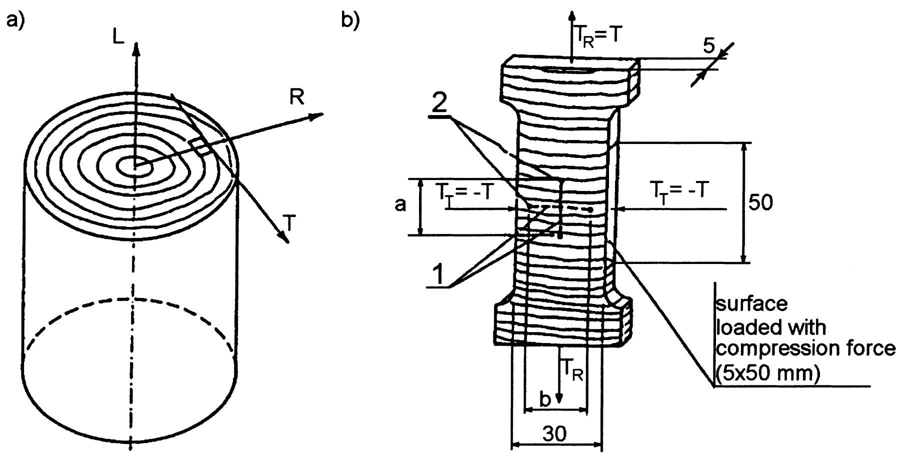

Figure 2. (a) Anatomic directions of wood (L-lengthwise, R-radial, T-tangential); (b) Wood sample (1-gauge length, $a = b = 20$mm, 2-gilding copper pins).

We introduce the engineering material constants $\mathcal{E}_i$ (Young modulus), $\mathcal{G}_{ij}$ (shear modulus), ν_{ij} (Poisson's ratio) and write the equation (3.6) in a developed form for the orthotropic model (see Lechnicky, 1977):

$$\begin{aligned}
E_{\mathrm{R}} &= \overset{\circ}{E}_{\mathrm{R}}(\Theta, \vartheta) + \frac{1}{\mathcal{E}_{\mathrm{R}}} T_{\mathrm{R}} - \frac{\nu_{\mathrm{TR}}}{\mathcal{E}_{\mathrm{T}}} T_{\mathrm{T}} - \frac{\nu_{\mathrm{LR}}}{\mathcal{E}_{\mathrm{L}}} T_{\mathrm{L}}, \\
E_{\mathrm{T}} &= \overset{\circ}{E}_{\mathrm{T}}(\Theta, \vartheta) - \frac{\nu_{\mathrm{RT}}}{\mathcal{E}_{\mathrm{R}}} T_{\mathrm{R}} + \frac{1}{\mathcal{E}_{\mathrm{T}}} T_{\mathrm{T}} - \frac{\nu_{\mathrm{LT}}}{\mathcal{E}_{\mathrm{L}}} T_{\mathrm{L}} \\
E_{\mathrm{L}} &= \overset{\circ}{E}_{\mathrm{L}}(\Theta, \vartheta) - \frac{\nu_{\mathrm{RT}}}{\mathcal{E}_{\mathrm{R}}} T_{\mathrm{R}} - \frac{\nu_{\mathrm{TL}}}{\mathcal{E}_{\mathrm{T}}} T_{\mathrm{T}} + \frac{1}{\mathcal{E}_{\mathrm{L}}} T_{\mathrm{L}}, \\
E_{\mathrm{RT}} &= \frac{1}{2\mathcal{G}_{\mathrm{RT}}} T_{\mathrm{RT}}, \qquad E_{\mathrm{RL}} = \frac{1}{2\mathcal{G}_{\mathrm{RL}}} T_{\mathrm{RL}}, \qquad E_{\mathrm{TL}} = \frac{1}{2\mathcal{G}_{\mathrm{TL}}} T_{\mathrm{TL}}.
\end{aligned} \tag{3.7}$$

The material constants have to satisfy the symmetry conditions:

$$\frac{\nu_{\mathrm{TR}}}{\mathcal{E}_{\mathrm{T}}} = \frac{\nu_{\mathrm{RT}}}{\mathcal{E}_{\mathrm{R}}}, \qquad \frac{\nu_{\mathrm{LR}}}{\mathcal{E}_{\mathrm{L}}} = \frac{\nu_{\mathrm{RL}}}{\mathcal{E}_{\mathrm{R}}}, \qquad \frac{\nu_{\mathrm{TL}}}{\mathcal{E}_{\mathrm{T}}} = \frac{\nu_{\mathrm{LT}}}{\mathcal{E}_{\mathrm{L}}}, \tag{3.8}$$

Due to this symmetry the number of material constants is reduced by three.

4. Experimental Studies: Estimation of Material Constants

The experiments were carried out on specially prepared samples of pine sapwood. The procedure of sample preparation and detailed methods of testing were presented in the paper by Kowal *et al.* (1992). Three states of stress were investigated, namely: tension in radial direction, compression in tangential direction, and both of the above states of stress acting together (biaxial stress). The individual stresses of value 1.00 MPa, 0.75 MPa, 0.50 MPa were applied to the samples according to the following programmes of wetting:

– constant moisture content $\Theta = 1\%$ (dry sample),
– varying moisture content from 1 to 36%,
– constant moisture content $\Theta \geqslant 36\%$.

In the second programm of wetting the samples were moisted in steps reaching in successive phases average values 1%, 7%, 14%, 22%, 29% and 36% of moisture content within 30 min. Changes in the gauge length of the measure basis, see Figure 2(b), were controlled every minute. Similarly, the strains of dry sample ($\Theta = 1\%$) and fully wet sample ($\Theta \geqslant 36\%$) were observed for 30 min, however, the changes in the gauge length were insignificant in these cases. Thus, the creep strains were negligibly small.

The thickness of the samples in the fibre lengthwise direction was 5 mm. The strain E_{L} in this direction can be considered as negligibly small for the above mentioned states of stress.

In all our experimental tests $T_L = 0$, $T_{RL} = 0$, $T_{TL} = 0$. Thus, we had tested the plane state of stress and the physical relation (3.7) for such a state reduce themselves:

$$E_R(T_R, T_T, \Theta, \vartheta) = \overset{\circ}{E}_R(\Theta, \vartheta) + \frac{1}{\mathcal{E}_R(\Theta, \vartheta)} T_R - \frac{V_{TR}}{\mathcal{E}_T(\Theta, \vartheta)} T_T, \tag{4.1}$$

$$E_T(T_R, T_T, \Theta, \vartheta) = \overset{\circ}{E}_T(\Theta, \vartheta) - \frac{V_{RT}}{\mathcal{E}_R(\Theta, \vartheta)} T_R + \frac{1}{\mathcal{E}_T(\Theta, \vartheta)} T_T, \tag{4.2}$$

$$E_{RT}(T_{RT}, \Theta, \vartheta) = \frac{1}{2\mathcal{G}_{RT}(\Theta, \vartheta)} T_{RT}. \tag{4.3}$$

It is well known that wood changes its mechanical properties in the range from dry state to the state of fiber saturation point. The swelling takes place only in this range, and the moisture content in wood is described by the isotherms of sorption. The isotherms of sorption (see, e.g., Ościk, 1982); Scheidegger, 1957; Kneule, 1970) indicate clearly that the moisture content is a non-linear function of the air relative humidity. According to the Langmuir theory (see, e.g., Kneule, 1970) the relation between the moisture content in wood Θ and the relative humidity of the surrounding atmosphere φ is

$$\Theta = \Theta_m \frac{c\varphi}{1 + c\varphi} = \Theta_m c\varphi[1 - (c\varphi) + (c\varphi)^2 - (c\varphi)^3 + \cdots], \tag{4.4}$$

where Θ_m is the moisture content when the adsorbent surface becomes saturated with a monomolecular adsorbate layer, c is a constant, and $0 \leqslant \varphi \leqslant 1$.

Assuming the swelling strain to be a function of moisture content one arrives at a non-linear relation of the form

$$\overset{\circ}{E}_i(\Theta, \vartheta) = a_i^{(0)} = a_i^{(0)} + a_i^{(1)} X + a_i^{(2)} X^2 + a_i^{(3)} X^3 + \cdots \tag{4.5}$$

for $i = \{R, T\}$. In the above relation $a_i^{(k)}$ denote some constants, and

$$X = (\Theta - \Theta_0)/(\Theta_n - \Theta_0), \quad 0 \leqslant X \leqslant 1, \tag{4.6}$$

is the relative moisture content in wood; Θ_0 denotes the initial saturation and Θ_n is the moisture content at the fibre saturation point.

An analysis of the experimental results indicates that the swelling strains reach their maximum values at the fibre saturation point Θ_n and additionally the function $\overset{\circ}{E}_T(\Theta, \vartheta)$ has the point of inflection. Thus, the free swelling strains have to fulfil the following conditions:

$$\overset{\circ}{E}_i(\Theta, \vartheta) = \overset{\circ}{E}_i(\Theta_0, \vartheta), \quad \text{for } X = 0, \tag{4.7a}$$

$$\overset{\circ}{E}_i(\Theta, \vartheta) = \overset{\circ}{E}_i(\Theta_n, \vartheta), \quad \text{for } X = 1, \tag{4.7b}$$

$$\frac{\partial \overset{\circ}{E}_i(\Theta, \vartheta)}{\partial \Theta} = 0, \quad \text{for } X = 1, \tag{4.7c}$$

$$\frac{\partial^2 \overset{\circ}{E}_{\rm T}(\Theta,\vartheta)}{\partial\Theta^2} = 0, \quad \text{in the point of inflection,} \tag{4.7d}$$

and

$$\frac{\partial^3 \overset{\circ}{\mathcal{E}}_{\rm T}(\Theta,\vartheta)}{\partial\Theta^3} < 0. \tag{4.7e}$$

We decided to approximate the free swelling strain in radial direction with a quadratic polynomial (no point of inflection) and that in tangential direction with a cubic polynomial. Estimating the constants $a_i^{(k)}$ in (4.5) on the basis of condition (4.6) and the method of quadratures one finds

$$\overset{\circ}{E}_{\rm R}(X,\vartheta) = \overset{\circ}{E}_{\rm R}(\Theta_\circ,\vartheta) + [\overset{\circ}{E}_{\rm R}(\Theta_{\rm n},\vartheta) - \overset{\circ}{E}_{\rm R}(\Theta_\circ,\vartheta)](2X - X^2), \tag{4.8}$$

$$\overset{\circ}{E}_{\rm T}(X,\vartheta) = \overset{\circ}{E}_{\rm T}(\Theta_\circ,\vartheta) + [\overset{\circ}{E}_{\rm T}(\Theta_{\rm n},\vartheta) - \overset{\circ}{E}_{\rm T}(\Theta_\circ,\vartheta)](3X^2 - 2X^3), \tag{4.9}$$

where $\overset{\circ}{E}_{\rm R}(X,\vartheta) \equiv \overset{\circ}{E}_{\rm R}(\Theta,\vartheta)$ and $\overset{\circ}{E}_{\rm T}(X,\vartheta) \equiv \overset{\circ}{E}_{\rm T}(\Theta,\vartheta)$.

The experimental curves and their theoretical interpolations for free swelling strains are presented in Figure 3. The root mean square deviation is about 5%.

4.1. ESTIMATION OF MATERIAL COEFFICIENTS

4.1.1. *Test 1*

In the first test the tension in radial direction was applied, that is, in the direction perpendicular to the annual rings. The radial stress applied to the sample was $T_{\rm R} = T = \{0.50, 0.75, 1.00\}$ MPa, and $T_{\rm T}$ and $T_{\rm TR}$ were equal to zero. The measured quantities in this test were radial $E'_{\rm R}$ and tangential $E'_{\rm T}$ strains. Thus, we could calculate:

$$\mathcal{E}_{\rm R}(\Theta,\vartheta) = \frac{T}{E'_{\rm R}}, \quad \text{and} \quad \nu_{\rm RT} = -\frac{E'_{\rm T}}{E'_{\rm R}}, \tag{4.10}$$

where $E'_{\rm R} = E_{\rm R}(T,0,\Theta,\vartheta) - \overset{\circ}{E}_{\rm R}(\Theta,\vartheta)$ and $E'_{\rm T} = E_{\rm T}(T,0,\Theta,\vartheta) - \overset{\circ}{E}_{\rm T}(\Theta,\vartheta)$.

Next, the coefficients $\mathcal{E}_{\rm R}(\Theta_\circ,T)$ and $B_{\rm R}(\Theta_\circ,T)$ were estimated in the following relation:

$$\frac{1}{\mathcal{E}_{\rm R}(X,\vartheta)} = \frac{1}{\mathcal{E}_{\rm R}(\Theta_\circ,\vartheta)} + B_{\rm R}(\Theta_\circ,\vartheta)X, \tag{4.11}$$

where $\mathcal{E}_{\rm R}(X,\vartheta) \equiv \mathcal{E}_{\rm R}(\Theta,\vartheta)$.

The Poisson's ratio is assumed to be constant, $\nu_{\rm RT} \cong 0.31$. The values of $\mathcal{E}_{\rm R}(\Theta_\circ, \vartheta) \cong 715$ MPa and $B_{\rm R}(\Theta_\circ,\vartheta) \cong 0.0022\ {\rm MPa}^{-1}$.

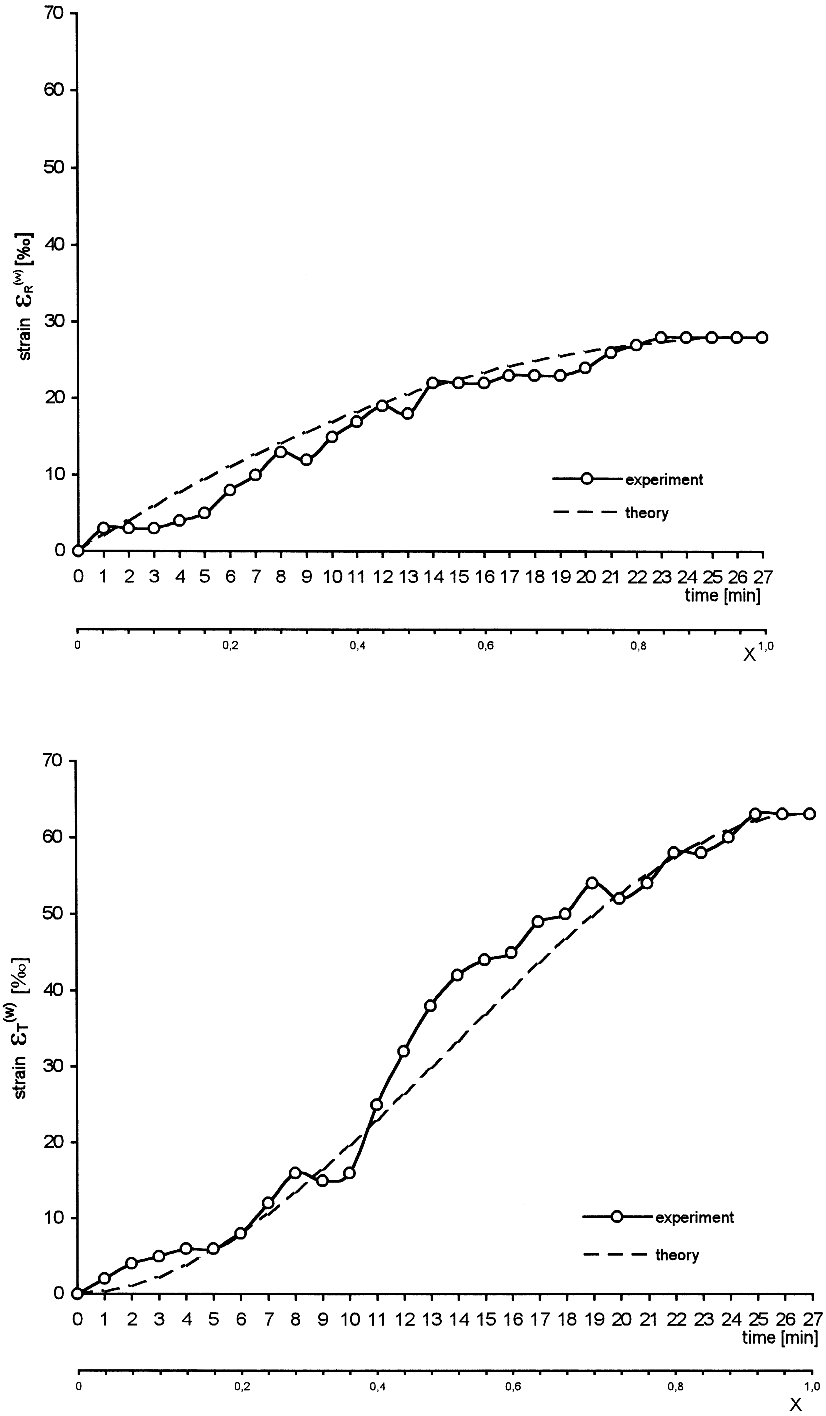

Figure 3. Swelling strains of the unstressed wood in radial E_R and tangential E_T direction.

4.1.2. *Test 2*

In this test $T_{\mathrm{T}} = -T = \{0.50, 0.75, 1.00\}$ MPa and the other stresses were zero. The measured quantities were radial E''_{R} and tangential E''_{T} strains. Young modulus $\mathcal{E}_{\mathrm{T}}$ and Poisson's ratio ν_{TR} were calculated, that is

$$\mathcal{E}_{\mathrm{T}}(\Theta, \vartheta) = \frac{|T|}{|E''_{\mathrm{T}}|}, \quad \text{and} \quad \nu_{\mathrm{RT}} = -\frac{E''_{\mathrm{R}}}{E''_{\mathrm{T}}}, \tag{4.12}$$

where

$$E''_{\mathrm{T}} = E_{\mathrm{T}}(0, -T, \Theta, \vartheta) - \mathring{E}_{\mathrm{T}}(\Theta, \vartheta) \quad \text{and} \quad E''_{\mathrm{R}} = E_{\mathrm{R}}(0, -T, \Theta, \vartheta) - \mathring{E}_{\mathrm{R}}(\Theta, \vartheta).$$

In the following the coefficients, $\mathcal{E}_{\mathrm{T}}(\Theta_\circ, \vartheta)$ and, $B_{\mathrm{T}}(\Theta_\circ, \vartheta)$ were estimated in the relation

$$\frac{1}{\mathcal{E}_{\mathrm{T}}(X, \vartheta)} = \frac{1}{\mathcal{E}_{\mathrm{T}}(\Theta_\circ, \vartheta)} + B_{\mathrm{T}}(\Theta_\circ, \vartheta)X, \tag{4.13}$$

where $\mathcal{E}_{\mathrm{T}}(X, \vartheta) \equiv \mathcal{E}_{\mathrm{T}}(\Theta, \vartheta)$.

The Poisson's ratio is assumed to be constant, $\nu_{\mathrm{TR}} \cong 0.25$. The values of $\mathcal{E}_{\mathrm{T}}(\Theta_\circ, \vartheta) \cong 562$ MPa and $B_{\mathrm{T}}(\Theta_\circ, \vartheta) \cong 0.030\ \mathrm{MPa}^{-1}$.

4.1.3. *Test 3*

In this test $T_{\mathrm{R}} = T$, $T_{\mathrm{T}} = -T$, $T = \{0.50, 0.75, 1.00\}$ MPa and $T_{\mathrm{RT}} = |T_{\mathrm{R}} - T_{\mathrm{T}}|/2 = T$. The measured quantity is $E'''_{\mathrm{RT}} = (E'''_{\mathrm{R}} - E'''_{\mathrm{T}})$. In this way we calculate the shear modulus:

$$\mathcal{G}_{\mathrm{RT}}(\Theta, \vartheta) = \frac{T_{\mathrm{RT}}}{2E'''_{\mathrm{RT}}} = \frac{T}{E'''_{\mathrm{R}} - E'''_{\mathrm{T}}}, \tag{4.14}$$

where

$$E'''_{\mathrm{R}} = E_{\mathrm{R}}(T, -T, \Theta, \vartheta) - \mathring{E}_{\mathrm{R}}(\Theta, \vartheta) \quad \text{and} \quad E'''_{\mathrm{T}} = E_{\mathrm{T}}(T, -T, \Theta, \vartheta) - \mathring{E}_{\mathrm{T}}(\Theta, \vartheta),$$

and construct the relation

$$\frac{1}{\mathcal{G}_{\mathrm{RT}}(X, \vartheta)} = \frac{1}{\mathcal{G}_{\mathrm{RT}}(\Theta_\circ, \vartheta)} + B_{\mathrm{RT}}(\Theta_\circ, \vartheta)X, \tag{4.15}$$

where $\mathcal{G}_{\mathrm{RT}}(X, \vartheta) \equiv \mathcal{G}_{\mathrm{RT}}(\Theta, \vartheta)$. The coefficients take the values of $\mathcal{G}_{\mathrm{RT}}(\Theta_\circ, \vartheta) = 245$ MPa and $B_{\mathrm{RT}}(\Theta_\circ, \vartheta) = 0.048$ MPa.

Now, we can write the final form of physical relations for the biaxial state of stress:

$$\begin{aligned} &E_{\mathrm{R}}(T_{\mathrm{R}}, T_{\mathrm{T}}, X, \vartheta) \\ &\quad = \mathring{E}_{\mathrm{R}}(\Theta_{\circ}, \vartheta) + [\mathring{E}_{\mathrm{R}}(\Theta_{\mathrm{n}}, \vartheta) - \mathring{E}_{\mathrm{R}}(\Theta_{\circ}, \vartheta)](2X - X^2) + \\ &\quad + \left[\frac{T_{\mathrm{R}}}{\mathcal{E}_{\mathrm{R}}(\Theta_{\circ}, \vartheta)} + B_{\mathrm{R}}(\Theta_{\circ}, \vartheta) \cdot X \cdot T_{\mathrm{R}}\right] - \\ &\quad - \nu_{\mathrm{TR}}\left[\frac{T_{\mathrm{T}}}{\mathcal{E}_{\mathrm{T}}(\Theta_{\circ}, \vartheta)} + B_{\mathrm{T}}(\Theta_{\circ}, \vartheta) \cdot X \cdot T_{\mathrm{T}}\right], \end{aligned} \tag{4.16}$$

$$\begin{aligned} E_{\mathrm{T}}(T_{\mathrm{R}}, T_{\mathrm{T}}, X, \vartheta) \;=\; & \mathring{E}_{\mathrm{T}}(\Theta_{\circ}, \vartheta) + [\mathring{E}_{\mathrm{T}}(\Theta_{\mathrm{n}}, \vartheta) - \\ & - \mathring{E}_{\mathrm{T}}(\Theta_{\circ}, \vartheta)](3X^2 - 2X^3) \; + \\ & + \nu_{\mathrm{RT}}\left[\frac{T_{\mathrm{R}}}{\mathcal{E}_{\mathrm{R}}(\Theta_{\circ}, \vartheta)} + B_{\mathrm{R}}(\Theta_{\circ}, \vartheta) \cdot X \cdot T_{\mathrm{R}}\right] - \\ & - \left[\frac{T_{\mathrm{T}}}{\mathcal{E}_{\mathrm{T}}(\Theta_{\circ}, \vartheta)} + B_{\mathrm{T}}(\Theta_{\circ}, \vartheta) \cdot X \cdot T_{\mathrm{T}}\right], \end{aligned} \tag{4.17}$$

$$2E_{\mathrm{RT}}(T_{\mathrm{RT}}, X, \vartheta) = \frac{T_{\mathrm{RT}}}{\mathcal{G}_{\mathrm{RT}}(\Theta_{\circ}, \vartheta)} + B_{\mathrm{RT}}(\Theta_{\circ}, \vartheta) \cdot X \cdot T_{\mathrm{RT}}. \tag{4.18}$$

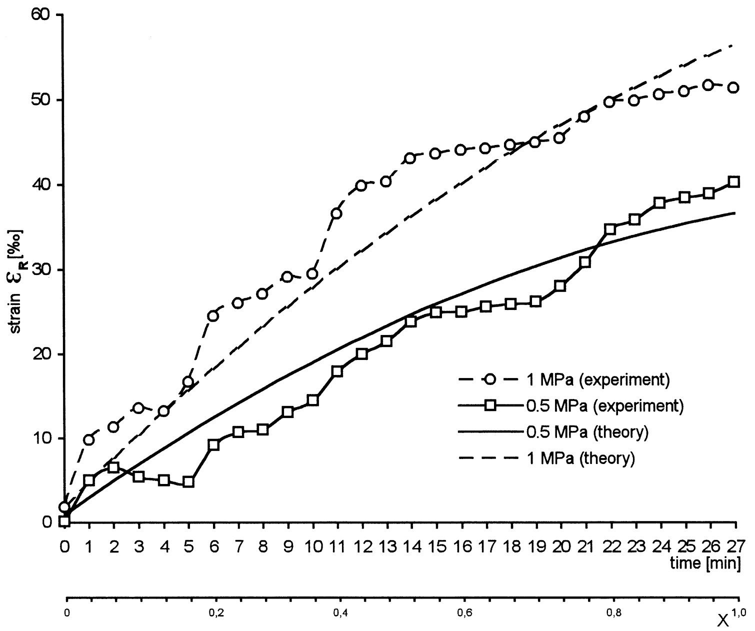

Figure 4. Swelling strains in the radial rection $\mathrm{E_R}$ of the sample, both tensed in radial direction $\mathrm{T_R = T}$ and compressed in tangential direction $\mathrm{T_T = -T}$.

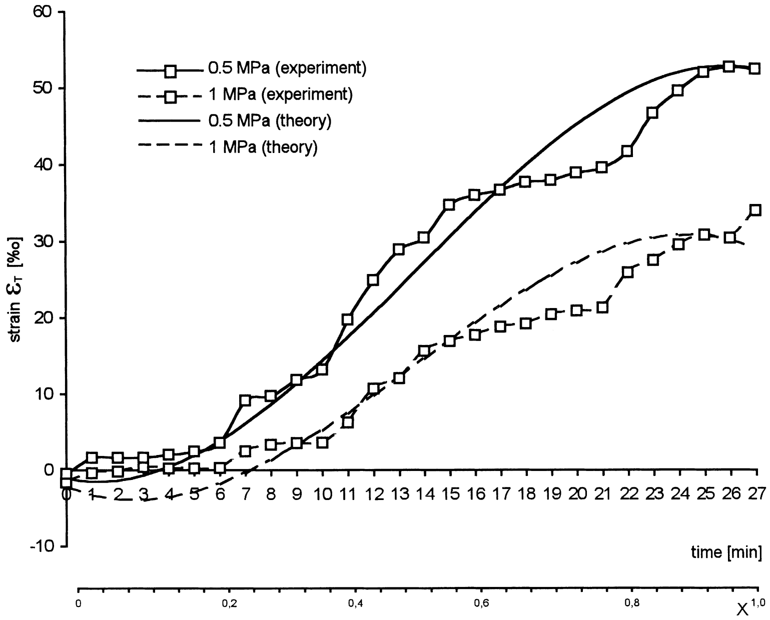

Figure 5. Swelling strains in the tangential direction E_T of the sample, both tensed in radial direction $T_R = T$ and compressed in tangential direction $T_T = -T$.

The material constants in the first two of the above equations were established in Test 1 and Test 2. A question arises whether these theoretical equations fit the experimental data obtained in Test 3. Figures 4 and 5 illustrate the compliance of the theoretical curves with those obtained from experiments.

The main common feature of both theoretical and experimental curves is their divergence for various stress values with increase of the moisture content. This phenomenon is due to coupling of mechanical state with sorption process. Other words, the mechanical properties of wood vary in the course of wetting process and the material coefficients depend evidently on the moisture content.

5. Conclusions

The main achievement of this paper are the physical relations for wet wood, whose wetting process was carried out in the stressed state. The so-called mechano-sorptive effect, observed in experimental tests (see Kowal *et al.*, 1992), has been taken into account in these relations. The mechano-sorptive strain in our theory has been assumed to be proportional to the product of stress and moisture content. It arose because the material coefficients were assumed as being linear functions of the moisture content. This term causes the curves of strain-moisture content to be not parallel with each other for various stresses.

The mechano-sorptive effect arises due to the simultaneous action of stress and wetting process. There is no such phenomenon when the sample is first wetted and then loaded. In the latter case, there is only the free swelling strain $\overset{\circ}{E}_i(\Theta,)$ and the instantaneous strain $T_i/\mathcal{E}_i(\Theta)$ with Young‘s modulus suitable for the actual moisture content Θ, $i = \mathrm{R}, \mathrm{T}$.

We are aware that the real wood is more complex than we have modelled it here and that we have neglected in our considerations a number of its important features. For example, assuming the orthotropic model for wood, we have ignored the differences between young and older layers of the annual rings, the curvature of these rings, variation of the structure alongside the height and the diameter of the trunk, and any defects of wood structure. This model, however, is a good approximation for wooden elements, whose dimensions are small in comparison with the size of the trunk (see Molinski and Raczkowski, 1982). In small samples, the curvature of annual rings becomes unnoticable, the sections of these rings are seen almost as straight elements perpendicular to the radial plain (see Figure 2(b)). Also the plane perpendicular to the axis of the trunk can be considered perpendicular to all the fibres of the trunk.

According to Leontiew (1952) studies concerning the pine wood, the modulus of elasticity when stretched along the fibres, were decreased in the direction towards the top by 27.7% along the trunk section of 7.7 m, that is barely 0.18% per 5 cm.

A relatively biggest approximation is in assuming the tangential plane to be a plane of elastic symmetry. This is because the elastic properties of wood in radial directions are highly differentiated not only because of the various properties of the layers of early and old wood in annual rings but also because of different thickness of these rings. According to Keylwerth (1951), studies of the modulus of elasticity in radial, tangential, and sloping at the angle of 45° to the radial direction have the following proportions: 1:0.52:0.22.

In spite of the simplifications we have made in our considerations, one can say that the mechano-sorptive effect is a universal phenomena for every wood undergoing a wetting process at the presence of a state of stress. In our opinion the mechano-sorptive strain presented in this paper, being proportional to the product of stress and moisture content, is a good approximation of the real coupling phenomena resulting from wetting and simultaneous loading of wood.

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