Minimizing the weight of a flooring strip: a shape optimization approach.

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GIREF

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We are a group of researchers mostly from Laval University in Quebec city more precisely:

- GIREF: Multidisciplinary finite element research center ,
- CRB: Wood research center
- Forintek: Wood Products Research Institute .

Characteristics of our activities:

- development of mathematical model for wood and wood composites,
- development of simulation tools for wood and wood composites,
- numerics mainly based on the finite element method,
- emphasis on experimental validation.

The numerical strategies are based on an object-oriented (c++) library called MEF++ developed by the GIREF over the past years.

This presentation stem from an ongoing general analysis on various aspects of thermo-hydromechanical behavior of flooring strip.

The parts considered here are generic flooring strips (hardwood floor, engeneering wood floor, etc).

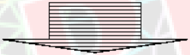
A study of the effect of the shape of grooves underneath a flooring strip lead us to consider the value of the different strip with respect to a weight reduction.

We are interested in diminishing their weight for two main reasons:

- better use of the resources (particle board, pellet, etc)
- improved shipping and handling

Regarding the optimization of a flooring strip:

- the apparent side cannot be modified.
- dimensional stability (deformations) must be considered
- the “shape” of the strip has an impact on its dimensional stability.
- Except for pathological case the deformation will be caused by thermal and hygrometric variations: warping.



- The reduction of the weight can be achieve by cutting grooves underneath the flooring strip.
- The reduction of the weight cannot be done without taking into account the dimensional stability.

Hypothesis for the model (Deteix et al, W & F Sc. , 2008)

- Transient moisture movement described by a 3-D conservation equation,
- for the displacements a linear orthotropic elastic model applies, there is no mechano-sorptive effects (Blanchet 2003)
- isothermal condition applies, the flooring strip is assumed to be initially free of stress.
- water vapor adsorption/desorption occur by free convection
- the stiffness and shrinking/swelling coefficients are constant
- hysteresis in adsorption/desorption for the shrinking/swelling coefficients (Goulet-Fortin 1975)
- no external forces are applied to the flooring strip: free standing or fastened.

Let be M the moisture content (moisture conservation eq.)

basal density d_b,
effective conductivity K_M
convective mass transfer h_M
ambiant moisture content M_∞

$$\frac{d_b}{100}\frac{\partial M}{\partial t} - \nabla \cdot (K_M \nabla M) = 0$$

$$q_M = \nabla \cdot (K_M \nabla M) \cdot n = h_M (M - M_\infty) \quad \text{on } \Gamma_{ex}$$

$$M = M_0 \qquad \text{at } t = 0$$

and U the displacements (quasi-static, small perturbations):

$\epsilon_{ij}(U) = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$,
stiffness tensor E_{ijkl}
swelling/shrinking tensor β_{ij}
stress tensor σ

$$-\nabla \cdot (E_{ijkl}(\epsilon_{ij}(\Delta U) - \beta_{ij}(\Delta M))) = 0$$

$$U_\alpha = 0 \text{ on } \Gamma^\alpha_{mec} \quad \alpha = x, y, z$$

$$U = 0 \quad \sigma(U) = 0 \quad \text{at } t = 0$$

Γ_{ex}: Top of the strip with moisture flux (none elsewhere).
Γ_{mec}: boundary part needed for the fastening or the freestanding B.C.

First approach: two basic “discrete” design optimization

Using the warping as a comparative tool we can analyze the merit of a finite (discrete) number of designs relative to some hygrometric variations.

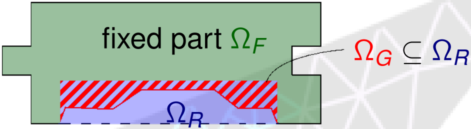
- case 1 The best design will minimise the deformations within a bounded weight reduction of the flooring strip.
- case 2 The best design will reduce the weight for within an acceptable range of deformations of the flooring strip.

Since we have a finite number of designs an hygro-mechanical analysis of each design is possible.

The discrete approach gives biassed results since we have a pre-established set of configurations of shape and number of grooves. Therefore we have no guarantee that the result is the most favorable.

We are interested in avoiding the subjectivity of the discrete design optimization. To do so we will have to

- consider all possible configurations (shape and number) of grooves,
- establish as much as possible an unbiased and non local measure of warping



whole strip : $\Omega = \Omega_F \cup \Omega_G \subseteq \Omega_F \cup \Omega_R = \Omega_{max}$

“Expand” the set of grooves: any 3D subset of a fixed “research zone” will be considered. Infinite number of admissible grooves!

- Finding the best design with respect to warping cannot be done by a simple analysis for each design.

A shape optimization or optimal design problem is of course the best suited approach.

- However for such a systematic approach a precise definition of the warping becomes essential.

The 4 basic types of warping are:


Twist


bow


crook


cup

Considering only the cupping can lead to bad design: a design which minimise cupping but increase the other types of warping.

We use the strain energy $S(t, \Omega_G)$ as a global indication of warping. To remove the time dependancy we take the maximal value (over time) of the strain energy as our measure of the warping:

$$C(\Omega_G) = \max_{t \geq 0} S(t, \Omega_G)$$

The moisture component of the strain correspond to an external force:

$$S(t,\Omega_G) = -\int_\Omega \nabla\cdot(E_{ijkl} : \beta_{ij}\Delta M)\cdot U\, d\Omega = -\int_\Omega (E_{ijkl} : \beta_{ij})\nabla M\cdot U\, d\Omega$$

Let t_{max} be the time at which the strain energy is maximal for Ω_{max}

Let M_{max} be the moisture content for Ω_{max} and $M^\star = M_{max}(t_{max})$

When $t \le t_{max}$, because of the boundary conditions, the variation of M can be neglected in Ω_R. Thus

$\forall\Omega_G \subseteq \Omega_R \quad M(t) = M_{max}(t),\ \forall t \le t_{max},$ and

$$C(\Omega_G) = S(t_{max},\Omega_G) = -\int_\Omega (E_{ijkl} : \beta_{ij})\nabla M^\star\cdot U\, d\Omega$$

The weight of the part is considered at a reference moisture content M_{ref}. We define the weight as a function of the basal density of the strip:

$$W(\Omega_G) = 1000. * \int_{\Omega} \frac{0.3 * d_b * (100 + M_{ref})}{30 - 0.265 * (30 - M_{ref}) * d_b} \, d\Omega$$

Maximizing the stiffness/ Minimizing the weight

$$\min_{\Omega_G \subseteq \Omega_R} C(\Omega_G)$$

such that

$$W(\Omega_G) \leq \alpha W(\Omega_R)$$

$$\min_{\Omega_G \subseteq \Omega_R} W(\Omega_G)$$

such that

$$C(\Omega_G) \leq (1+\alpha) C(\Omega_R)$$

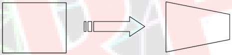
- $\alpha \in [0, 1]$
- Other geometrical or mechanical conditions:
  - locally bounded displacements
  - locally bounded average stress
  - bounds on ponctual measure of cupping
  - etc

Not limited to flooring strip: those formulations are very general and can be used in other context. We could consider other admissible shapes, cost functional and mechanical boundary conditions.

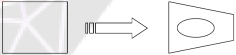
Shape optimization: finding the geometry of a domain that will minimize (or maximise) a given criteria (cost function, objective function) under certain constraint.

The two main branches of shape optimization:

- Variation of the boundaries: the optimal shape is determined by perturbation of the boundaries of an initial shape.



- Topological optimization: no a priori knowledge of the shape: we can generate ”holes“ and hollow shapes.



The variation of the boundaries correspond to a descent method: creation a minimizing sequence (of shape) using the shape gradient of the cost function J.

Shape derivative: Let Ω_0 be an initial shape and V a vector field. The Hadamard-Zolesio theorem states the shape gradient $dJ(\Omega)$ satisfy:

$$\lim_{t \to 0} \frac{J((I + tV)\Omega_0) - J(\Omega_0)}{t} = dJ(\Omega_0) \cdot V = \int_{\partial\Omega_0} j'(\Omega_0) V \cdot n \, ds$$

For $V_i = -j'(\Omega_i)n \quad i = 0, \dots$ and $\alpha > 0$ small, we have (*formally*)

$$J((I + \alpha V_i)\Omega_i) \approx J(\Omega_i) - \alpha \int_{\partial\Omega_i} (j'(\Omega_i))^2 \, ds < J(\Omega_i)$$

and we have a diminishing sequence of shape: $\Omega_{i+1} = (I + \alpha V)\Omega_i$

- Since the shape gradient depends only on the boundary we cannot generate ”holes“ or modify the topology in any way.
- The calculation of the shape gradient can be difficult (but not in our case). It could involve the solution of an adjoint problem.
- Since the domain change at every step we will have to ”remesh“. Moreover local minima are unavoidable.
- The method can be applied to a large number of problem and can easily produce ”smooth“ solution depending on the definition of the boundary (NURBS, spline, piecewise constant, ...)

Topological optimization correspond to finding the distribution of void and material in a fixed domain D (in our case D correspond to Ω_{max}).

Let ρ be the volume fraction of material,

$$\rho \in X(D) = \{\chi \in L^{\infty}(D) |\, \chi(x) \in \{0,1\} a.e.\ in D\}$$

$$\text{finding } \Omega \subseteq D \longleftrightarrow \text{finding } \rho \in X(D)$$

$X(D)$ is not compact nor convex: the solution can be outside of $X(D)$.

One way of insuring the existence of a solution is to enlarge $X(D)$

$$X(D) = \{\rho \in L^{\infty}(D) |\, \rho(x) \in [0,1]\}.$$

Then we have weak compactness but the solution present micro-structure associated with the intermediate values.

Two main ways of dealing with the ”intermediate“ zone:

- construct composite material for intermediate volume fraction
- penalize the use of intermediate volume fraction

SIMP = Solid (Simple) Isotropic Microstructure (Material) Penalized

The SIMP method consist in penalizing the intermediates values with a power law:

$$E_{ijkl}(\rho) = \rho^p E_{ijkl} \quad p \geq 1$$

- it is simple to implement, computationally efficient.
- mesh dependancy and checkerboard effects are eliminate by filtering technique.
- one scalar unknown produce ”rough“ solution
- local minima are unavoidable but we use only one mesh

Both of these optimization technique have their inconvenients. One interesting approach is to combine them:

- step 1 Solve the topological optimization problem. This allows us to generate ”voids“. The calculation will be done on a single finite element mesh. Can be numerically difficult (possibly a non cenvex optimization problem).
- step 2 Using the solution to the topological problem as an initial shape we apply the variation of the boundaries technique. This will give us a refined solution. We should note however that this method is numerically expensive since it requires frequent remeshing.

Speed method SIMP method Alogorithme general contenant des condtions geometric et mecanique supplementaires

Definition du probleme contraintes supplementaire possibles resultats dans le cas free-standing resultats preliminaire dans le cas cloue

Lame de parquets et autres applications: framing, coffins, etc.