CRM Summer School on Multiphysics Solvers: An introduction to Domain Decomposition Methods

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The module serves as an introduction to domain decomposition (DD) solvers and preconditioners. The following outline describes the content of the first three lectures, scheduled for Wed June 2nd (morning and afternoon) and Thu June 3rd (morning only). Each lecture is 1.5 hours long (split in 2 parts of 45 minutes each). The fourth lecture of this mini-course will be dedicated to a short tutorial on a new domain decomposition library `ffddm`[1]. The exercise session will be dedicated to the important key topics in the theory and implementation of the methods. All the methods and algorithms will be illustrated by numerical examples and simulations. References to past and current work will be indicated for those who want to deepen the theoretical part.

Prerequisites and resources: A part of the lecture will follow a few chapters of the book "An introduction to domain decomposition methods: algorithms, theory and parallel implementation", SIAM 2015. This book is freely downloadable together with the Freefem codes used to illustrate the methods:
`http://www.victoritadolean.com/p/book.html`.
Installation of the open source software Freefem (`https://freefem.org/`) is also required beforehand. (available for all platforms both in binary and compiled version)

Lecture 1: A basic introduction to DD methods

`Objectives`: This presentation will be kept at a basic level, both continuous and algebraic versions of the methods will be given in their most common variants and the main ingredients of domain decomposition methods will be presented. The content will follow the lines of the chapters 1 and 3 from the domain decomposition book. A short introduction to Freefem software will be given which will allow the students to use quickly the codes illustrating the methods.

`Outcomes`: At the end of this first lecture, students will have a basic understanding of the methods but also of their implementation. Depending on the progress on this first part, some concepts might be recalled and/or continued in the second lecture.

Lecture 2: Two-level methods

`Objectives`: Domain decomposition methods are meant to be used as parallel solvers and scalability (behaviour independent of the number of subdomains/processors) and robustness with respect to the physical parameters are very important issues. An introduction to coarse spaces

[1] `https://doc.freefem.org/documentation/ffddm/index.html`

and two-level methods for symmetric positive definite (SPD) problems will be given together with the presentation of a few variants of domain decomposition preconditioners (AS, RAS, ORAS, SORAS). The content will follow chapters 4 and 5 from the book, although more recent research results will also be included.

`Outcomes`: Students will be able to understand the use and the impact of the two-level methods both for scalability and robustness (even if at this stage the codes are sequential).

Lecture 3: Robust solvers for time-harmonic wave propagation problems

`Objectives`: Numerical simulation for wave propagation problems presents various challenges at different levels and is motivated by various applications arising in electrical engineering or geophysics. Designing robust fast solvers is far more complicated since solutions are highly oscillatory requiring fine discretisations which lead usually to very large linear systems to solve. These systems are often indefinite or involving non-normal matrices hence most of the usual iterative solvers will struggle to converge to the solution and the convergence will deteriorate with the wave-number. This topic is part of a larger research project during the past years and during this lecture we will see a few adaptations which are necessary when dealing with wave propagation problems: reference to the codes and recent research papers will be provided.

`Outcomes`: Understand the main differences between the usual SPD problems and more complex problems and how these differences will translate into the design of the domain decomposition methods.